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**Stopping Distance** The research and development department of an automobile manufacturer has determined that when required to stop quickly to avoid an accident, the distance (in feet) a car travels during the driver's reaction time is given by  $R(x) = \frac{3}{4}x$ , where  $x$  is the speed of the car in miles per

hour. The distance (in feet) traveled while the driver is braking is given by  $B(x) = \frac{1}{15}x^2$ .

- Find the function that represents the total stopping distance  $T$ .
- Use a graphing utility to graph the functions  $R$ ,  $B$ , and  $T$  in the same viewing window for  $0 \leq x \leq 60$ .
- Which function contributes most to the magnitude of the sum at higher speeds? Explain.

**Ripples** A pebble is dropped into a calm pond, causing ripples in the form of concentric circles (see figure). The radius (in feet) of the outer ripple is given by  $r(t) = 0.6t$ , where  $t$  is the time (in seconds) after the pebble strikes the water. The area of the circle is given by  $A(r) = \pi r^2$ . Find and interpret  $(A \circ r)(t)$ .

**Geometry** A square concrete foundation was prepared as a base for a large cylindrical gasoline tank (see figure).

- Write the radius  $r$  of the tank as a function of the length  $x$  of the sides of the square.
- Write the area  $A$  of the circular base of the tank as a function of the radius  $r$ .
- Find and interpret  $(A \circ r)(x)$ .

**Cost** The weekly cost  $C$  of producing  $x$  units in a manufacturing process is given by  $C(x) = 60x + 750$ . The number of units  $x$  produced in  $t$  hours is  $x(t) = 50t$ .

- Find and interpret  $(C \circ x)(t)$ .
- Use a graphing utility to graph the cost as a function of time. Use the trace feature to estimate (to two-decimal-place accuracy) the time that must elapse until the cost increases to \$15,000.

**Air Traffic Control** An air traffic controller spots two planes at the same altitude flying toward each other. Their flight paths form a right angle at point  $P$ . One plane is 150 miles from point  $P$  and is moving at 450 miles per hour. The other plane is 200 miles from point  $P$  and is moving at 450 miles per hour. Write the distance  $s$  between the planes as a function of time  $t$ .

**Salary** You are a sales representative for an automobile manufacturer. You are paid an annual salary plus a bonus of 3% of your sales over \$500,000. Consider the two functions  $f(x) = x - 500,000$  and  $g(x) = 0.03x$ .

If  $x$  is greater than \$500,000, which of the following represents your bonus? Explain.

- $f(g(x))$
- $g(f(x))$

**Consumer Awareness** The suggested retail price of a new car is  $p$  dollars. The dealership advertised a factory rebate of \$1200 and an 8% discount.

- Write a function  $R$  in terms of  $p$  giving the cost of the car after receiving the rebate from the factory.
- Write a function  $S$  in terms of  $p$  giving the cost of the car after receiving the dealership discount.
- Form the composite functions  $(R \circ S)(p)$  and  $(S \circ R)(p)$  and interpret each.
- Find  $(R \circ S)(18400)$  and  $(S \circ R)(18400)$ . Which yields the lower cost for the car? Explain.

**Transportation** The total value of new car sales  $f$  (in billions of dollars) in the United States from 1995 through 2001 is shown in the table. The time (in years) is given by  $t$ , with  $t = 5$  corresponding to 1995.

- Does  $f^{-1}$  exist?

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- (b) If  $f^{-1}$  exists, what does it mean in the context of the problem?
- (c) If  $f^{-1}$  exists, find  $f^{-1}(650.3)$ .
- (d) If the table above were extended to 2002 and if the total value of new car sales for that year were \$546.3 billion, would  $f^{-1}$  exist? Explain.

**Hourly Wage** Your wage is \$8.00 per hour plus \$0.75 for each unit produced per hour. So, your hourly wage  $y$  in terms of the number of units produced is  $y = 8 + 0.75x$ .

- (a) Find the inverse function. What does each variable in the inverse function represent?
- (b) Use a graphing utility to graph the function and its inverse function.
- (c) Use the trace feature of a graphing utility to find the hourly wage when 10 units are produced per hour.
- (d) Use the trace feature of a graphing utility to find the number of units produced when your hourly wage is \$22.25.

**Hooke's Law** Hooke's Law states that the force  $F$  required to compress or stretch a spring (within its elastic limits) is proportional to the distance  $d$  that the spring is compressed or stretched from its original length. That is,  $F = kd$ , where  $k$  is the measure of the stiffness of the spring and is called the spring constant. The table shows the elongation  $d$  in centimeters of a spring when a force of  $F$  kilograms is applied.

- (a) Sketch a scatter plot of the data.
- (b) Find the equation of the line that seems to best fit the data.
- (c) Use the regression feature of a graphing utility to find a linear model for the data. Compare this model with the model from part (b).
- (d) Use the model from part (c) to estimate the elongation of the spring when a force of 55 kilograms is applied.

**Radio** The number  $R$  of U.S. radio stations for selected years from 1970 through 2000 is shown in the table.

- (a) Use the regression feature of a graphing utility to find a linear model for the data. Let  $t$  represent the year, with  $t = 0$  corresponding to 1970.
- (b) Use a graphing utility to plot the data and graph the model in the same viewing window.
- (c) Interpret the slope of the model in the context of the problem.
- (d) Use the model to predict the number of radio stations in 2010.

**Cost** A hand tool manufacturer produces a product for which the variable cost is \$5.35 per unit and the fixed costs are \$16,000. The company sells the product for \$8.20 and can sell all that it produces.

- (a) Write the total cost  $C$  as a function of  $x$ , the number of units produced.
- (b) Write the profit  $P$  as a function of  $x$ .

Find two positive real numbers whose product is a maximum.

- (a) The sum is 110.
- (b) The sum is  $S$ .
- (c) The sum of the first and twice the second is 24.
- (d) The sum of the first and three times the second is 42.

**Geometry** An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter single-lane running track.

- (a) Draw a diagram that illustrates the problem. Let  $x$  and  $y$  represent the length and width of the rectangular region, respectively.
- (b) Determine the radius of the semicircular ends of the track. Determine the distance, in terms of  $y$ , around the inside edge of the two semicircular parts of the track.

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- (c) Use the result of part (b) to write an equation, in terms of  $x$  and  $y$ , for the distance traveled in one lap around the track. Solve for  $y$ .
- (d) Use the result of part (c) to write the area  $A$  of the rectangular region as a function of  $x$ .
- (e) Use a graphing utility to graph the area function from part (d). Use the graph to approximate the dimensions that will produce a rectangle of maximum area.

**Numerical, Graphical, and Analytical Analysis** A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). Use the following methods to determine the dimensions that will produce a maximum enclosed area.

- (a) Write the area  $A$  of the corral as a function of  $x$ .
- (b) Use the table feature of a graphing utility to create a table showing possible values of  $x$  and the corresponding areas of the corral. Use the table to estimate the dimensions that will produce the maximum enclosed area.
- (c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce the maximum enclosed area.
- (d) Write the area function in standard form to find algebraically the dimensions that will produce the maximum area.
- (e) Compare your results from parts (b), (c), and (d).

**Height of a Ball** The height  $y$  (in feet) of a ball thrown by a child is given by

$$y = -\frac{1}{12}x^2 + 2x + 4$$

where  $x$  is the horizontal distance (in feet) from where the ball is thrown (see figure).

- (a) Use a graphing utility to graph the path of the ball.
- (b) How high is the ball when it leaves the child's hand? (Hint: Find  $y$  when  $x = 0$ .)
- (c) What is the maximum height of the ball?
- (d) How far from the child does the ball strike the ground?

**Path of a Diver** The path of a diver is given by

$$y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$$

where  $y$  is the height (in feet) and  $x$  is the horizontal distance (in feet) from the end of the diving board (see figure). What is the maximum height of the diver? Verify your answer using a graphing utility.

**Cost** A manufacturer of lighting fixtures has daily production costs of

$$C = 800 - 10x + 0.25x^2$$

where  $C$  is the total cost (in dollars) and  $x$  is the number of units produced. Use the table feature of a graphing utility to determine how many fixtures should be produced each day to yield a minimum cost.

**Revenue** The total revenue  $R$  (in millions of dollars) for a company is related to its advertising expense by the function  $R = 0.00001(-x^3 + 600x^2)$ ,  $0 \leq x \leq 400$ , where  $x$  is the amount spent on advertising (in tens of thousands of dollars). Use the graph of the function shown in the figure to estimate the point on the graph at which the function is increasing most rapidly. This point is called the point of diminishing returns because any expense above this amount will yield less return per dollar invested in advertising.

**Height** A baseball is thrown upward from ground level with an initial velocity of 48 feet per second, and its height  $h$  (in feet) is given by

$$h(t) = -16t^2 + 48t, 0 \leq t \leq 3$$

where  $t$  is the time (in seconds). You are told that the ball reaches a height of 64 feet. Is this possible? Explain.

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**Profit** The demand equation for a microwave is  $p = 140 - 0.0001x$ , where  $p$  is the unit price (in dollars) of the microwave and  $x$  is the number of units produced and sold. The cost equation for the microwave is  $C = 80x + 150,000$ , where  $C$  is the total cost (in dollars) and  $x$  is the number of units produced. The total profit obtained by producing and selling  $x$  units is given by

$$P = R - C = xp - C.$$

You are working in the marketing department that produces this microwave, and you are asked to determine a price  $p$  that would yield a profit of \$9 million. Is this possible? Explain.

**Concentration of a Mixture** A 1000-liter tank contains 50 liters of a 25% brine solution. You add  $x$  liters of a 75% brine solution to the tank.

(a) Show that the concentration  $C$ , the proportion of brine to the total solution, of the final mixture is given by

$$C = \frac{3x + 50}{4(x + 50)}$$

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Use a graphing utility to graph the function. As the tank is filled, what happens to the rate at which the concentration of brine increases? What percent does the concentration of brine appear to approach?

**Geometry** A rectangular region of length  $x$  and width  $y$  has an area of 500 square meters.

(a) Write the width  $y$  as a function of  $x$ .

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Sketch a graph of the function and determine the width of the rectangle when  $x = 30$  meters.

**Page Design** A page that is  $x$  inches wide and  $y$  inches high contains 30 square inches of print. The margins at the top and bottom are 2 inches deep and the margins on each side are 1 inch wide (see figure).

(a) Show that the total area  $A$  of the page is given by  $A = \frac{2x(2x + 11)}{x - 2}$

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Use a graphing utility to graph the area function and approximate the page size such that the minimum amount of paper will be used. Verify your answer numerically using the table feature of a graphing utility.

**Geometry** A right triangle is formed in the first quadrant by the  $x$ -axis, the  $y$ -axis, and a line segment through the point  $(3, 2)$  (see figure).

(a) Show that an equation of the line segment is given by  $y = \frac{2(a - x)}{a - 3}$

(b) Show that the area of the triangle is given by  $A = \frac{a^2}{a - 3}$

(c) Use a graphing utility to graph the area function and estimate the value of  $a$  that yields a minimum area. Estimate the minimum area. Verify your answer numerically using the table feature of a graphing utility.

**Cost** The ordering and transportation cost  $C$  (in thousands of dollars) for the components used in manufacturing a product is given by

$$C = 100 \left( \frac{200}{x^2} + \frac{x}{x + 30} \right)$$

where  $x$  is the order size (in hundreds). Use a graphing utility to graph the cost function. From the graph, estimate the order size that minimizes cost.

**Average Cost** The cost  $C$  of producing  $x$  units of a product is given by  $C = 0.2x^2 + 10x + 5$ , and the average cost per unit is given by

$$\bar{C} = \frac{C}{x} = \frac{0.2x^2 + 10x + 5}{x}$$

Sketch the graph of the average cost function, and estimate the number of units that should be produced to minimize the average cost per unit.

**Medicine** The concentration  $C$  of a chemical in the bloodstream  $t$  hours after injection into muscle tissue is given by

$$C = \frac{3t^2 + t}{t^3 + 50}$$

- Determine the horizontal asymptote of the function and interpret its meaning in the context of the problem.
- Use a graphing utility to graph the function and approximate the time when the bloodstream concentration is greatest.
- Use a graphing utility to determine when the concentration is less than 0.345.

**Demand** The demand function for a product is given by  $p = 5000 \left( 1 - \frac{4}{4 + e^{-0.002x}} \right)$

where  $p$  is the price and  $x$  is the number of units.

- Use a graphing utility to graph the demand function for  $x > 0$  and  $p > 0$ .
- Find the price  $p$  for a demand of  $x = 500$  units.
- Use the graph in part (a) to approximate the highest price that will still yield a demand of at least 600 units.
- Verify your answers to parts (b) and (c) numerically by creating a table of values for the function.

**Compound Interest** There are three options for investing \$500. The first earns 7% compounded annually, the second earns 7% compounded quarterly, and the third earns 7% compounded continuously.

- Find equations that model each investment growth and use a graphing utility to graph each model in the same viewing window over a 20-year period.
- Use the graph from part (a) to determine which investment yields the highest return after 20 years. What is the difference in earnings between each investment?

**Radioactive Decay** Let  $Q$  represent a mass of radioactive radium ( $^{226}\text{Ra}$ ), in grams, whose half-life is 1620 years. The quantity of radium present after  $t$  years is given by  $Q = 25 \left( \frac{1}{2} \right)^{t/1620}$ .

- Determine the initial quantity (when  $t = 0$ ).
- Determine the quantity present after 1000 years.
- Use a graphing utility to graph the function over the interval  $t = 0$  to  $t = 5000$ .
- When will the quantity of radium be 0 grams? Explain.

**Population Growth** The population of a town increases according to the model  $P(t) = 2500e^{0.0293t}$ , where  $t$  is the time in years, with  $t = 0$  corresponding to 2000.

- Use a graphing utility to graph the function for the years 2000 through 2025.
- Use a graphing utility to approximate the population in 2015 and 2025.
- Verify your answers in part (b) algebraically.

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**Inflation** If the annual rate of inflation averages 4% over the next 10 years, the approximate cost  $C$  of goods or services during any year in that decade will be modeled by  $C(t) = P(1.04)^t$ , where  $t$  is the time (in years) and  $P$  is the present cost. The price of an oil change for your car is presently \$23.95.

- Use a graphing utility to graph the function.
- Use the graph in part (a) to approximate the price of an oil change 10 years from now.
- Verify your answer in part (b) algebraically.

**Human Memory Model** Students in a mathematics class were given an exam and then tested monthly with an equivalent exam. The average scores for the class are given by the human memory model

$$f(t) = 80 - 17\log_{10}(t + 1), 0 \leq t \leq 12$$

where  $t$  is the time in months.

- What was the average score on the original exam ( $t = 0$ )?
- What was the average score after 4 months?
- What was the average score after 10 months?
- Verify your answers in parts (a), (b), and (c) using a graphing utility.

**Demand** The demand equation for a camera is given by  $p = 500 - 0.5(e^{0.004x})$ . Find the demand  $x$  for a price of (a)  $p = \$350$  and (b)  $p = \$300$ .

**Forestry** The number of trees per acre  $N$  of a certain species is approximated by the model

$$N = 68(10^{-0.04x}), 5 \leq x \leq 40$$

where  $x$  is the average diameter of the trees (in inches) three feet above the ground. Use the model to approximate the average diameter of the trees in a test plot for which  $N = 21$ .

**Bacteria Growth** The number  $N$  of bacteria in a culture is given by the model  $N = 100e^{kt}$  where  $t$  is the time (in hours). If  $N = 300$  when  $t = 5$ , estimate the time required for the population to double in size. Verify your estimate graphically.

**Radioactive Decay** The half-life of radioactive radium ( $^{226}\text{Ra}$ ) is 1620 years. What percent of a present amount of radioactive radium will remain after 100 years?

**Carbon Dating** Carbon 14 ( $^{14}\text{C}$ ) dating assumes that the carbon dioxide on Earth today has the same radioactive content as it did centuries ago. If this is true, the amount of  $^{14}\text{C}$  absorbed by a tree that grew several centuries ago should be the same as the amount of  $^{14}\text{C}$  absorbed by a tree growing today. A piece of ancient charcoal contains only 15% as much radioactive carbon as a piece of modern charcoal. How long ago was the tree burned to make the ancient charcoal if the half-life of  $^{14}\text{C}$  is 5730 years.

**Depreciation** A sport utility vehicle (SUV) that cost \$32,000 new has a book value of \$18,000 after 2 years.

- Find the linear model  $V = mt + b$ .
- Find the exponential model  $V = ae^{kt}$ .
- Use a graphing utility to graph the two models in the same viewing window. Which model depreciates faster in the first year?
- Use each model to find the book values of the SUV after 1 year and after 3 years.
- Interpret the slope of the linear model.

**Depreciation** A computer that cost \$2000 new has a book value of \$500 after 2 years.

- Find the linear model  $V = mt + b$ .
- Find the exponential model  $V = ae^{kt}$ .
- Use a graphing utility to graph the two models in the same viewing window. Which model depreciates faster in the first year?

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- (d) Use each model to find the book values of the computer after 1 year and after 3 years.  
(e) Interpret the slope of the linear model.

**Sales** The sales  $S$  (in thousands of units) of a new CD burner after it has been on the market  $t$  years are given by  $S = 100(1 - e^{kt})$ . Fifteen thousand units of the new product were sold the first year.

- (a) Complete the model by solving for  $k$ .  
(b) Use a graphing utility to graph the model.  
(c) Use the graph in part (b) to estimate the number of units sold after 5 years.

**Sales** The sales  $S$  (in thousands of units) of a cleaning solution after  $x$  hundred dollars is spent on advertising are given by  $S = 10(1 - e^{kx})$

When \$500 is spent on advertising, 2500 units are sold.

- (a) Complete the model by solving for  $k$ .  
(b) Estimate the number of units that will be sold if advertising expenditures are raised to \$700.

As a result of the installation of a muffler, the noise level of an engine was reduced from 88 to 72 decibels. Find the percent decrease in the intensity level of the noise due to the installation of the muffler.

Find the pH if  $[H^+] = 2.3 \times 10^{-5}$ .

Compute  $[H^+]$  for a solution for which  $\text{pH} = 5.8$ .

A grape has a pH of 3.5, and milk of magnesia has a pH of 10.5. The hydrogen ion concentration of the grape is how many times that of the milk of magnesia?

The pH of a solution is decreased by one unit. The hydrogen ion concentration is increased by what factor?

**Newton's Law of Cooling** At 8:30 A.M., a coroner was called to the home of a person who had died during the night. In order to estimate the time of death, the coroner took the person's temperature twice. At 9:00 A.M. the temperature was  $85.7^\circ\text{F}$ , and at 11 :00 A.M. the temperature was  $82.8^\circ\text{F}$ . From these two temperatures the coroner was able to determine that the time elapsed since death and the body temperature were related by the formula

$$t = -\ln \frac{T - 70}{98.6 - 70}$$

where  $t$  is the time (in hours elapsed since the person died) and  $T$  is the temperature (in degrees Fahrenheit) of the person's body. Assume that the person had a normal body temperature of  $98.6^\circ\text{F}$  at death and that the room temperature was a constant  $70^\circ\text{F}$ . Use the formula to estimate the time of death of the person.

**Difference in Latitudes** Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Syracuse, New York and Annapolis, Maryland, where Syracuse is 450 kilometers due north of Annapolis?

**Difference in Latitudes** Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Lynchburg, Virginia and Myrtle Beach, South Carolina, where Lynchburg is 400 kilometers due north of Myrtle Beach?

**Instrumentation** A voltmeter's pointer is 6 centimeters in length (see figure). Find the angle through which it rotates when it moves 2.5 centimeters on the scale.

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**Electric Hoist** An electric hoist is used to lift a piece of equipment 2 feet (see figure). The diameter of the drum on the hoist is 10 inches. Find the number of degrees through which the drum must rotate.

**Sports** The number of revolutions made by a figure skater for each type of axel jump is given. Determine the measure of the angle generated as the skater performs each jump. Give the answer in both degrees and radians.

(a) Single axel:  $1\frac{1}{2}$  (b) Double axel:  $2\frac{1}{2}$  (c) Triple axel:  $3\frac{1}{2}$

**Linear Speed** A satellite in circular orbit 1250 kilometers above Earth makes one complete revolution every 110 minutes. What is its linear speed? Assume that Earth is a sphere of radius 6400 kilometers.

**Construction** The circular blade on a saw has a diameter of 7.5 inches and rotates at 2400 revolutions per minute (see figure).

(a) Find the angular speed in radians per second.

(b) Find the linear speed of the saw teeth (in feet per second) as they contact the wood being cut.

**Angular Speed** A car is moving at a rate of 40 miles per hour, and the diameter of its wheels is 2.5 feet.

(a) Find the number of revolutions per minute the wheels are rotating.

(b) Find the angular speed of the wheels in radians per minute.

**Electric Circuits** The initial current and charge in an electrical circuit are zero. The current when 100 volts is applied to the circuit is given by  $I = 5e^{-2t}\sin t$  where the resistance, inductance, and capacitance are 80 ohms, 20 henrys, and 0.01 farad, respectively.

Approximate the current (in amperes)  $t = 0.7$  second after the voltage is applied.

**Harmonic Motion** The displacement from equilibrium of an oscillating weight suspended by a spring is given by

$$y(t) = \frac{1}{4} \cos 6t$$

where  $y$  is the displacement in feet and  $t$  is the time in seconds (see figure). Find the displacement when (a)  $t = 0$ , (b)  $t = 4$ , and (c)  $t = 1/2$ .

**Harmonic Motion** The displacement from equilibrium of an oscillating weight suspended by a spring and subject to the damping effect of friction is given by  $y(t) = e^{-t}\cos 6t$ , where  $y$  is the displacement (in feet) and  $t$  is the time (in seconds). Find the displacement when (a)  $t = 0$ , (b)  $t = 1/4$  and (c)  $t = 1/2$

**Average Cost** The average cost  $C$  (in dollars per pound) of recycling a waste product  $X$  (in pounds) is given by

$$\bar{C} = \frac{450000 + 5x}{x}$$

Find the average cost of recycling  $x = 10,000$  pounds,  $x = 100,000$  pounds, and  $x = 1,000,000$  pounds. According to this model, what is the limiting average cost as the number of pounds increases?

**Inflation** If the inflation rate averages 4.5% over the next 10 years, the approximate cost  $C$  of goods or services  $t$  years from now is given by  $C(t) = P(1.045)^t$  where  $P$  is the present cost. The price of a tire is presently \$69.95. Estimate the price 10 years from now.

**Height** A six-foot person walks from the base of a streetlight directly toward the tip of the shadow cast by the streetlight. When the person is 16 feet from the streetlight and 5 feet from the tip of the streetlight's shadow, the person's shadow starts to appear beyond the streetlight's shadow.

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- (a) Draw a right triangle that gives a visual representation of the problem. Show the known quantities of the triangle and use a variable to indicate the height of the streetlight.
- (b) Use a trigonometric function to write an equation involving the unknown quantity.
- (c) What is the height of the streetlight?

**Height** A 30-meter line is used to tether a helium-filled balloon. Because of a breeze, the line makes an angle of approximately  $75^\circ$  with the ground.

- (a) Draw a right triangle that gives a visual representation of the problem. Show the known quantities of the triangle and use a variable to indicate the height of the balloon.
- (b) Use a trigonometric function to write an equation involving the unknown quantity.
- (c) What is the height of the balloon?

**Width** A biologist wants to know the width  $w$  of a river in order to properly set instruments for studying the pollutants in the water. From point A, the biologist walks downstream 100 feet and sights to point C. From this sighting, it is determined that  $\angle C = 58^\circ$ . How wide is the river? Verify your result numerically.

**Height of a Mountain** In traveling across flat land you notice a mountain directly in front of you. Its angle of elevation (to the peak) is  $3.5^\circ$ . After you drive 13 miles closer to the mountain, the angle of elevation is  $9^\circ$  (see figure). Approximate the height of the mountain.

- Angle of Elevation** A ramp 20 feet in length rises to a loading platform that is 3 feet off the ground.
- (a) Draw a right triangle that gives a visual representation of the problem. Show the known quantities of the triangle and use a variable to indicate the angle of elevation of the ramp.
  - (b) Use a trigonometric function to write an equation involving the unknown quantity.
  - (c) Use a graphing utility to approximate the angle of elevation numerically.

**Jin Mao Building** You are standing 65 meters from the base of the Jin Mao Building in Shanghai, China. You estimate that the angle of elevation to the top of the 88th floor (sightseeing level) is  $80^\circ$ . What is the approximate height of the building? One of your friends is on the sightseeing level. What is the distance between you and your friend?

- Length** A guywire is stretched from the top of a 200-foot broadcasting tower to an anchor making an angle of  $58^\circ$  with the ground.
- (a) How long is the wire?
  - (b) How far is the anchor from the base of the tower?

**Machine Shop Calculations** A tapered shaft has a diameter of 5 centimeters at the small end and is 15 centimeters long (see figure). The taper is  $3^\circ$ . Find the diameter  $d$  of the large end of the shaft.

**Machine Shop Calculations** A steel plate has the form of one fourth of a circle with a radius of 60 centimeters. Two two-centimeter holes are to be drilled in the plate, positioned as shown in the figure. Find the coordinates of the center of each hole.

**Geometry** Use a compass to sketch a quarter of a circle of radius 10 centimeters. Using a protractor, construct an angle of  $20^\circ$  in standard position (see figure). Drop a perpendicular from the point of intersection of the terminal side of the angle and the arc of the circle. By actual measurement, calculate the coordinates  $(x, y)$  of the point of intersection and use these measurements to approximate the six trigonometric functions of a  $20^\circ$  angle.

Determine the exact values of the six trigonometric functions of the angle  $\theta$ .

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**Distance** An airplane flying at an altitude of 6 miles is on a flight path that passes directly over an observer (see figure). If  $\theta$  is the angle of elevation from the observer to the plane. Find the distance from the observer to the plane when (a)  $\theta = 30^\circ$ , (b)  $\theta = 90^\circ$ , and (c)  $\theta = 120^\circ$ .

**Distance** A plane flying at an altitude of 5 miles over level ground will pass directly over a radar antenna (see figure). Let  $d$  be the ground distance from the antenna to the point directly under the plane and let  $x$  be the angle of elevation to the plane from the antenna. ( $d$  is positive as the plane approaches the antenna.) Write  $d$  as a function of  $x$  and graph the function over the interval  $0 < x < \pi$ .

**Television Coverage** A television camera is on a reviewing platform 36 meters from the street on which a parade will be passing from left to right (see figure). Write the distance  $d$  from the camera to a particular unit in the parade as a function of the angle  $x$ , and graph the function over the interval  $-\pi/2 < x < \pi/2$ . (Consider  $x$  as negative when a unit in the parade approaches from the left.)

**Predator-Prey Model** The population of coyotes (a predator) at time  $t$  (in months) in a region is estimated to be  $P = 10,000 + 3000\sin(\pi t/12)$  and the population of rabbits (its prey) is estimated to be  $p = 15,000 + 5000\cos(\pi t/12)$ . Use the graph of the models to explain the oscillations in the size of each population.

**Numerical and Graphical Reasoning** A crossed belt connects a 10-centimeter pulley on an electric motor with a 20-centimeter pulley on a saw arbor (see figure). The electric motor runs at 1700 revolutions per minute.

- Determine the number of revolutions per minute of the saw.
- How does crossing the belt affect the saw in relation to the motor?
- Let  $L$  be the total length of the belt. Write  $L$  as a function of  $\phi$ , where  $\phi$  is measured in radians. What is the domain of the function? (Hint: Add the lengths of the straight sections of the belt and the length of belt around each pulley.)
- Use a graphing utility to complete the table.
- As  $\phi$  increases, do the lengths of the straight sections of the belt change faster or slower than the lengths of the belts around each pulley?
- Use a graphing utility to graph the function over the appropriate domain.

Use the properties of inverse functions to find the exact value of the expression.  
 $\sin(\arcsin 0.7)$

**Docking a Boat** A boat is pulled in by means of a winch located on a dock 10 feet above the deck of the boat (see figure). Let  $\theta$  be the angle of elevation from the boat to the winch and let  $s$  be the length of the rope from the winch to the boat.

- Write  $\theta$  as a function of  $s$ .
- Find  $\theta$  when  $s = 52$  feet and when  $s = 26$  feet.

**Granular Angle of Repose** Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. This angle  $\theta$  is called the angle of repose. When rock salt is stored in a cone-shaped pile 11 feet high, the diameter of the pile's base is about 34 feet.

- Draw a diagram that gives a visual representation of the problem. Label all known and unknown quantities.
- Find the angle of repose for rock salt.
- How tall is a pile of rock salt that has a base diameter of 40 feet?

**Photography** A television camera at ground level is filming the lift-off of a space shuttle at a point 750 meters from the launch pad (see figure). Let  $\theta$  be the angle of elevation to the shuttle and let  $s$  be the height of the shuttle.

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- (a) Write  $\theta$  as a function of  $s$ .
- (b) Find  $\theta$  when  $s = 400$  meters and when  $s = 1600$  meters.

**Photography** A photographer is taking a picture of a three-foot painting hung in an art gallery. The camera lens is 1 foot below the lower edge of the painting (see figure). The angle  $\beta$  subtended by the camera lens  $x$  feet from the painting is

$$\beta = \arctan \frac{3x}{x^2 + 4}$$

- (a) Use a graphing utility to graph  $\beta$  as a function of  $x$ .
- (b) Move the cursor along the graph to approximate the distance from the picture when  $\beta$  is maximum.
- (c) Identify the asymptote of the graph and discuss its meaning in the context of the problem.

**Angle of Elevation** An airplane flies at an altitude of 6 miles toward a point directly over an observer. Consider  $\theta$  and  $x$  as shown in the figure.

- (a) Write  $\theta$  as a function of  $x$ .
- (b) Find  $\theta$  when  $x = 10$  miles and  $x = 3$  miles.

**Security Patrol** A security car with its spotlight on is parked 20 meters from a long warehouse. Consider  $\theta$  and  $x$  as shown in the figure.

- (a) Write  $\theta$  as a function of  $x$ .
- (b) Find  $\theta$  when  $x = 5$  meters and when  $x = 12$  meters.

**Length** A shadow of length  $L$  is created by a 60-foot silo when the sun is  $\theta^\circ$  above the horizon.

- (a) Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.
- (b) Write  $L$  as a function of  $\theta$ .
- (c) Use a graphing utility to complete the table.
- (d) The angle measure increases in equal increments in the table. Does the length of the shadow change in equal increments? Explain.

**Length** A shadow of length  $L$  is created by an 850-foot building when the sun is  $\theta^\circ$  above the horizon.

- (a) Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.
- (b) Write  $L$  as a function of  $\theta$ .
- (c) Use a graphing utility to complete the table.

**Height** A ladder 20 feet long leans against the side of a house. The angle of elevation of the ladder is  $80^\circ$ . Find the height from the top of the ladder to the ground.

**Height** The angle of elevation from the base to the top of a waterslide is  $13^\circ$ . The slide extends horizontally 58.2 meters. Approximate the height of the waterslide.

**Height** From a point 50 feet in front of a church, the angles of elevation to the base of the steeple and the top of the steeple are  $35^\circ$  and  $47^\circ 40'$ , respectively.

- (a) Draw right triangles that give a visual representation of the problem. Label the known and unknown quantities.
- (b) Use a trigonometric function to write an equation involving the unknown quantity.
- (c) Find the height of the steeple.

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**Height** From a point 100 feet in front of a public library, the angles of elevation to the base of the flagpole and the top of the flagpole are  $28^\circ$  and  $39^\circ 45'$ , respectively. The flagpole is mounted on the front of the library's roof. Find the height of the flagpole.

**Depth** The sonar of a navy cruiser detects a submarine that is 4000 feet from the cruiser. The angle between the water level and the submarine is  $31.5^\circ$ . How deep is the submarine?

**Height** A 100-foot line is attached to a kite. When the kite has pulled the line taut, the angle of elevation to the kite is approximately  $50^\circ$ . Approximate the height of the kite.

**Angle of Elevation** An engineer erects a 75-foot vertical cellular-phone tower. Find the angle of elevation to the top of the tower from a point on level ground 95 feet from its base.

**Angle of Elevation** The height of an outdoor basketball backboard is  $12\frac{1}{2}$  feet, and the backboard casts a shadow  $17\frac{1}{2}$  feet long.

- Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.
- Use a trigonometric function to write an equation involving the unknown quantity.
- Find the angle of elevation of the sun.

**Angle of Depression** A Global Positioning System satellite orbits 12,500 miles above Earth's surface. Find the angle of depression from the satellite to the horizon. Assume the radius of Earth is 4000 miles.

**Angle of Depression** Find the angle of depression from the top of a lighthouse 250 feet above water level to the water line of a ship  $2\frac{1}{2}$  miles offshore.

**Airplane Ascent** When an airplane leaves the runway, its angle of climb is  $18^\circ$  and its speed is 275 feet per second. Find the plane's altitude after 1 minute.

**Airplane Ascent** How long will it take the plane in Exercise 27 to climb to an altitude of 10,000 feet? 16,000 feet?

**Mountain Descent** A sign on the roadway at the top of a mountain indicates that for the next 4 miles the grade is  $9.5^\circ$  (see figure). Find the change in elevation for a car descending the mountain.

**Ski Slope** A ski slope on a mountain has an angle of elevation of  $25.2^\circ$ . The vertical height of the slope is 1808 feet. How long is the slope?

**Navigation** A ship leaves port at noon and has a bearing of  $S 29^\circ W$ . The ship sails at 20 knots. How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.?

**Navigation** An airplane flying at 600 miles per hour has a bearing of  $52^\circ$ . After flying 1.5 hours, how far north and how far east has the plane traveled from its point of departure?

**Surveying** A surveyor wants to find the distance across a swamp. The bearing from A to B is  $N 32^\circ W$ . The surveyor walks 50 meters from A, and at the point C the bearing to B is  $N 68^\circ W$ . Find (a) the bearing from A to C and (b) the distance from A to B.

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**Location of a Fire** Two fire towers are 30 kilometers apart, where tower A is due west of tower B. A fire is spotted from the towers, and the bearings from A and B are  $E 14^\circ N$  and  $W 34^\circ N$ , respectively. Find the distance  $d$  of the fire from the line segment AB.

**Navigation** A ship is 45 miles east and 30 miles south of port. The captain wants to sail directly to port. What bearing should be taken?

**Navigation** A plane is 160 miles north and 85 miles east of an airport. The pilot wants to fly directly to the airport. What bearing should be taken?

**Distance** An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are  $4^\circ$  and  $6.5^\circ$  (see figure). How far apart are the ships?

**Distance** A passenger in an airplane flying at an altitude of 10 kilometers sees two towns directly to the east of the plane. The angles of depression to the towns are  $28^\circ$  and  $55^\circ$  (see figure). How far apart are the towns?

**Altitude** A plane is observed approaching your home and you assume its speed is 550 miles per hour. The angle of elevation to the plane is  $16^\circ$  at one time and  $57^\circ$  one minute later. Approximate the altitude of the plane.

**Height** While traveling across flat land, you notice a mountain directly in front of you. The angle of elevation to the peak is  $2.5^\circ$ . After you drive 18 miles closer to the mountain, the angle of elevation is  $10^\circ$ . Approximate the height of the mountain.

**Geometry** Determine the angle between the diagonal of a cube and the diagonal of its base, as shown in the figure.

**Geometry** Determine the angle between the diagonal of a cube and its edge, as shown in the figure.

**Hardware** Write the distance  $y$  across the flat sides of a hexagonal nut as a function of  $r$ , as shown in the figure.

**Hardware** The figure shows a circular piece of sheet metal of diameter 40 centimeters. The sheet contains 12 equally spaced bolt holes. Determine the straight line distance between the centers of two consecutive bolt holes.

**Geometry** A regular pentagon (a pentagon with congruent sides and angles) is inscribed in a circle of radius 25 inches. Find the length of the sides of the pentagon.

**Geometry** A regular hexagon (a hexagon with congruent sides and angles) is inscribed in a circle of radius 25 inches. Find the length of the sides of the hexagon.

**Tuning Fork** A point on the end of a tuning fork moves in the simple harmonic motion described by  $d = a \sin \omega t$ . Find  $\omega$  given that the tuning fork for middle C has a frequency of 264 vibrations per second.

**Wave Motion** A buoy oscillates in simple harmonic motion as waves go past. At a given time it is noted that the buoy moves a total of 3.5 feet from its low point to its high point (see figure), and that it returns to its high point every 10 seconds. Write an equation that describes the motion of the buoy if it is at its high point at time  $t = 0$ .

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**Springs** A ball that is bobbing up and down on the end of a spring has a maximum displacement of 3 inches. Its motion (in ideal conditions) is modeled by  $y = \frac{1}{4} \cos 16t$  where  $y$  is measured in feet and  $t$  is the time in seconds.

- Use a graphing utility to graph the function.
- What is the period of the oscillations?
- Determine the first time the ball passes the point of equilibrium ( $y = 0$ ).

**Numerical and Graphical Analysis** A two-meter-high fence is 3 meters from the side of a grain storage bin. A grain elevator must reach from ground level outside the fence to the storage bin (see figure). The objective is to determine the shortest elevator that meets the constraints.

- Complete four rows of the table.
- Use the table feature of a graphing utility to generate additional rows of the table. Use the table to estimate the minimum length of the elevator.
- Write the length  $L_1 + L_2$  as a function of  $\theta$ .
- Use a graphing utility to graph the function. Use the graph to estimate the minimum length. How does your estimate compare with that in part (b)?

**Numerical and Graphical Analysis** The cross sections of an irrigation canal are isosceles trapezoids, where the length of three of the sides is 8 feet (see figure). The objective is to find the angle  $\theta$  that maximizes the area of the cross sections. [Hint: The area of a trapezoid is given by  $(h/2)(b_1 + b_2)$ .]

- Complete seven rows of the table.
- Use the table feature of a graphing utility to generate additional rows of the table. Use the table to estimate the maximum cross-sectional area.
- Write the area  $A$  as a function of  $\theta$ .
- Use a graphing utility to graph the function. Use the graph to estimate the maximum cross-sectional area. How does your estimate compare with that in part (b)?

**Width** An engineer is trying to determine the width of a river. From point P, the engineer walks downstream 125 feet and sights to point Q. From this sighting, it is determined that  $\theta = 62^\circ$ . How wide is the river?

**Height** An escalator 152 feet in length rises to a platform and makes a  $30^\circ$  angle with the ground.

- Draw a right triangle that gives a visual representation of the problem. Show the known quantities of the triangle and use a variable to indicate the height of the platform above the ground.
- Use a trigonometric function to write an equation involving the unknown quantity.
- Find the height of the platform above the ground.

**Railroad Grade** A train travels 3.5 kilometers on a straight track with a grade of  $1^\circ 10'$ . What is the vertical rise of the train in that distance?

**Mountain Descent** A road sign at the top of a mountain indicates that for the next 4 miles the grade is 12%. Find the angle of the grade and the change in elevation for a car descending the mountain.

**Distance** A passenger in an airplane flying at an altitude of 37,000 feet sees two towns directly to the west of the airplane. The angles of depression to the towns are  $32^\circ$  and  $76^\circ$  (see figure). How far apart are the towns?

**Distance** From city A to city B, a plane flies 650 miles at a bearing of  $48^\circ$ . From city B to city C, the plane flies 810 miles at a bearing of  $115^\circ$ . Find the distance from A to C and the bearing from A to C.

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**Wave Motion** A buoy oscillates in simple harmonic motion as waves go past. At a given time it is noted that the buoy moves a total of 6 feet from its low point to its high point, returning to its high point every 15 seconds (see figure). Write an equation that describes the motion of the buoy if it is at its high point at  $t = 0$ .

**Wave Motion** Your fishing bobber oscillates in simple harmonic motion from the waves in the lake where you fish. Your bobber moves a total of 1.5 inches from its high point to its low point and returns to its high point every 3 seconds. Write an equation modeling the motion of your bobber if it is at its high point at  $t = 0$ .

**Numerical Analysis** A 3000-pound automobile is negotiating a circular interchange of radius 300 feet at a speed of  $s$  miles per hour (see figure). The relationship between the speed and the angle  $\theta$  (in degrees) at which the roadway should be banked so that no lateral frictional force is exerted on the tires is  $\tan \theta = 0.672s^2/3000$ .

**Friction** The forces acting on an object weighing  $W$  units on an inclined plane positioned at an angle of  $\theta$  with the horizontal (see figure) are modeled by  $\mu W \cos \theta = W \sin \theta$  where  $\mu$  is the coefficient of friction. Solve the equation for  $\mu$  and simplify the result.

**Shadow Length** The length  $s$  of the shadow cast by a vertical gnomon (a device used to tell time) of height  $h$  when the angle of the sun above the horizon is  $\theta$  can be modeled by the equation

$$s = \frac{h \sin(90^\circ - \theta)}{\sin \theta}$$

Show that the equation is equivalent to  $s = h \cot \theta$ .

**Harmonic Motion** A weight is oscillating on the end of a spring (see figure). The position of the weight relative to the point of equilibrium is given by

$$y = \frac{1}{12}(\cos 8t - 3 \sin 8t)$$

where  $y$  is the displacement (in meters) and  $t$  is the time (in seconds). Find the times when the weight is at the point of equilibrium ( $y = 0$ ) for  $0 \leq t \leq 1$ .

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