1. A drug response curve describes the level of medication in the bloodstream after a drug is administered. A surge function $S(t) = At^p e^{-kt}$ is often used to model the response curve, reflecting an initial surge in the drug level and then a more gradual decline. If, for a particular drug, A = 0.01, p = 4, k = 0.07, and is measured in minutes, estimate the times corresponding to the inflection points and explain their significance. If you have a graphing device, use it to graph the drug response curve.

2. If the initial amount A_o of money is invested at an interest rate r compounded n times a years is

   $$A = A_o\left(1+\frac{r}{n}\right)^{nt}$$

   If we let $n \to \infty$, we refer to the continuous compounding of interest. Use l'Hopital's Rule to show that if interest is compounded continuously, then the amount after t years $A = A_o e^{rt}$.

3. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

4. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

5. If 1200 cm of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

6. A rectangular storage container with an open top is to have a volume of 10 m . The length of its base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

7. Find, correct to two decimal places, the coordinates of the point on the curve y=tan x that is closest to the point (1,1).

8. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

9. Find the area of the largest rectangle that can be inscribed in the ellipse $x^2/a^2 + y^2/b^2 = 1$.

10. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.

11. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y=8 - x^2$.

12. Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius r.

13. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs.

14. A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible volume of such a cylinder.

15. A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder.

16. A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible surface area of such a cylinder.

17. A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle. See Exercise 56 on page 23.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

18. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 cm^2 , find the dimensions of the poster with the smallest area.

19. A poster is to have an area of 180 in^2 with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?

20. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum?

21. A cylindrical can without a top is made to contain V cm^3 of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

22. A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

23. A cone-shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup.

24. A cone-shaped paper drinking cup is to be made to hold 27 cm^3 of water. Find the height and radius of the cup that will use the smallest amount of paper.

25. A cone with height is inscribed in a larger cone with height H so that its vertex is at the center of the base of the larger cone. Show that the inner cone has maximum volume when $h = \frac{1}{3}H$.

26. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle with a plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a constant called the coefficient of friction. For what value of θ is F smallest?

27. If a resistor of R ohms is connected across a battery of E volts with internal resistance r ohms, then the power (in watts) in the external resistor is

$$P = \frac{E^2 R}{(R+r)^2}$$

If E and r are fixed but R varies, what is the maximum value of the power?

28. For a fish swimming at a speed relative to the water, the energy expenditure per unit time is proportional to v^3. It is believed that migrating fish try to minimize the total energy required to swim a fixed distance. If the fish are swimming against a current u (u < v), then the time required to swim a distance L is L/(v – u) and the total energy required to swim the distance is given by

$$E(\text{v}) = a\text{v}^3 \cdot \frac{L}{\text{v} - u}$$

where a is the proportionality constant.

(a) Determine the value of v that minimizes E.

(b) Sketch the graph of E.

Note: This result has been verified experimentally; migrating fish swim against a current at a speed greater than the current speed.

29. In a beehive, each cell is a regular hexagonal prism, open at one end with a trihedral angle at the other end as in the figure. It is believed that bees form their cells in such a way as to minimize the surface area for a given volume, thus using the least amount of wax in cell construction. Examination of these cells has shown that the measure of the apex angle θ is amazingly consistent. Based on the geometry of the cell, it can be shown that the surface area S is given by

$$S = 6sh - \frac{3}{2}s^2 \cot\theta + (3s^2\sqrt{3}/2)\csc\theta$$

where s, the length of the sides of the hexagon, and h, the height, are constants.

(a) Calculate dS/dθ.

(b) What angle should the bees prefer?

(c) Determine the minimum surface area of the cell (in terms of s and h).

Note: Actual measurements of the angle θ in beehives have been made, and the measures of these angles seldom differ from the calculated value by more than 2°.

30. A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

31. A woman at a point A on the shore of a circular lake with radius 2 mi wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can walk at the rate of 4 mi/h and row a boat at 2 mi/h. How should she proceed?

32. An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 km east of the refinery. The cost of laying pipe is $400,000/km over land to a point P on the north bank and $800,000/km under the river to the tanks. To minimize the cost of the pipeline, where P should be located?

33. The illumination of an object by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources, one three

times as strong as the other, are placed 10 ft apart, where should an object be placed on the line between the sources so as to receive the least illumination?

34. Find an equation of the line through the point (3,5) that cuts off the least area from the first quadrant.

35. Let a and b be positive numbers. Find the length of the shortest line segment that is cut off by the first quadrant and passes through the point (a,b).

36. The illumination of an object by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources, one three times as strong as the other, are placed 10 ft apart, where should an object be placed on the line between the sources so as to receive the least illumination?

37. (a) If C(x) is the cost of producing units of a commodity, then the average cost per unit is c(x)=C(x)/x. Show that if the average cost is a minimum, then the marginal cost equals the average cost.

(b) If $C(x)=16000+200x+4x^{3/2}$, in dollars, find (i) the cost, average cost, and marginal cost at a production level of 1000 units; (ii) the production level that will minimize the average cost; and (iii) the minimum average cost.

38. (a) Show that if the profit P(x) is a maximum, then the marginal revenue equals the marginal cost.
(b) If $C(x)=16000+500x-1.6x^2+0.004x^3$ is the cost function and p(x)=1700-7x is the demand function, find the production level that will maximize profit.

39. A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000.
(a) Find the demand function, assuming that it is linear.
(b) How should ticket prices be set to maximize revenue?

40. During the summer months Terry makes and sells necklaces on the beach. Last summer he sold the necklaces for \$10 each and his sales averaged 20 per day. When he increased the price by \$1, he found that the average decreased by two sales per day.
(a) Find the demand function, assuming that it is linear.
(b) If the material for each necklace costs Terry \$6, what should the selling price be to maximize his profit?

41. A manufacturer has been selling 1000 television sets a week at \$450 each. A market survey indicates that for each \$10 rebate offered to the buyer, the number of sets sold will increase by 100 per week.
(a) Find the demand function.
(b) How large a rebate should the company offer the buyer in order to maximize its revenue?
(c) If its weekly cost function is C(x)=68,000+150x, how should the manufacturer set the size of the rebate in order to maximize its profit?

42. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?

43. Show that of all the isosceles triangles with a given perimeter, the one with the greatest area is equilateral.

44. The frame for a kite is to be made from six pieces of wood. The four exterior pieces have been cut with the lengths indicated in the figure. To maximize the area of the kite, how long should the diagonal pieces be?

45. A point P needs to be located somewhere on the line AD so that the total length L of cables linking P to the points A, B, and C is minimized (see the figure). Express L as a function of x=|AP| and use the graphs of L and dL/dx to estimate the minimum value.

46. The graph shows the fuel consumption c of a car (measured in gallons per hour) as a function of the speed v of the car. At very low speeds the engine runs inefficiently, so initially c decreases as the speed increases. But at high speeds the fuel consumption increases. You can see that c(v) is minimized for this car when v≈30 mi/h. However, for fuel efficiency, what must be minimized is not the consumption in gallons per hour but rather the fuel consumption in gallons per mile. Let's call this consumption G. Using the graph, estimate the speed at which G has its minimum value.

47. Let v_1 be the velocity of light in air and v_2 the velocity of light in water. According to Fermat's Principle, a ray of light will travel from a point A in the air to a point B in the water by a path ACB that minimizes the time taken. Show that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

where θ_1 (the angle of incidence) and θ_2 (the angle of refraction) are as shown. This equation is known as Snell's Law.

48. Two vertical poles PQ and ST are secured by a rope PRS going from the top of the first pole to a point R on the ground between the poles and then to the top of the second pole as in the figure. Show that the shortest length of such a rope occurs when $\theta_1=\theta_2$.

49. The upper right-hand corner of a piece of paper, 12 in. by 8 in., as in the figure, is folded over to the bottom edge. How would you fold it so as to minimize the length of the fold? In other words, how would you choose x to minimize y?

50. A steel pipe is being carried down a hallway 9 ft wide. At the end of the hall there is a right-angled turn into a narrower hallway 6 ft wide. What is the length of the longest pipe that can be carried horizontally around the corner?

51. An observer stands at a point P, one unit away from a track. Two runners start at the point S in the figure and run along the track. One runner runs three times as fast as the other. Find the maximum value of the observer's angle θ of sight between the runners.

52. A rain gutter is to be constructed from a metal sheet of width 30 cm by bending up one-third of the sheet on each side through an angle θ. How should θ be chosen so that the gutter will carry the maximum amount of water?

53. Where should the point P be chosen on the line segment AB so as to maximize the angle θ?

54. A painting in an art gallery has height h and is hung so that its lower edge is a distance d above the eye of an observer (as in the figure). How far from the wall should the observer stand to get the best view? (In other words, where should the observer stand so as to maximize the angle θ subtended at his eye by the painting?)

55. Find the maximum area of a rectangle that can be circumscribed about a given rectangle with length L and width W. [Hint: Express the area as a function of an angle θ.]

56. The blood vascular system consists of blood vessels (arteries, arterioles, capillaries, and veins) that convey blood from the heart to the organs and back to the heart. This system should work so as to minimize the energy expended by the heart in pumping the blood. In particular, this energy is reduced when the resistance of the blood is lowered. One of Poiseuille's Laws gives the resistance R of the blood as

$$R = C\frac{L}{r^4}$$

where L is the length of the blood vessel, r is the radius, and C is a positive constant determined by the viscosity of the blood. (Poiseuille established this law experimentally, but it also follows from Equation 8.4.2.) The figure shows a main blood vessel with radius r_1 branching at an angle θ into a smaller vessel with radius r_2.

(a) Use Poiseuille's Law to show that the total resistance of the blood along the path ABC is

$$R = C\left(\frac{a - b\cot\theta}{r_1^4} + \frac{b\csc\theta}{r_2^4}\right)$$

where a and b are the distances shown in the figure.

(b) Prove that this resistance is minimized when $\cos\theta = \frac{r_2^4}{r_1^4}$

(c) Find the optimal branching angle (correct to the nearest degree) when the radius of the smaller blood vessel is two-thirds the radius of the larger vessel.

57. Use Newton's method with initial approximation $x_1=-1$ to find x_2, the second approximation to the root of the equation $x^3+x+3=0$. Explain how the method works by first graphing the function and its tangent line at (-1,1).

58. Use Newton's method to find the coordinates, correct to six decimal places, of the point on the parabola $y=(x-1)^2$ that is closest to the origin.

59. A stone is dropped from the upper observation deck (the Space Deck) of the CN Tower, 450 m above the ground.

(a) Find the distance of the stone above ground level at time t.
(b) How long does it take the stone to reach the ground?
(c) With what velocity does it strike the ground?
(d) If the stone is thrown downward with a speed of 5 m/s, how long does it take to reach the ground?

60. Two balls are thrown upward from the edge of the cliff in Example 7. The first is thrown with a speed of 48 ft/s and the other is thrown a second later with a speed of 24 ft/s. Do the balls ever pass each other?

61. A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff?

62. A company estimates that the marginal cost (in dollars per item) of producing x items is 1.92-0.002x. If the cost of producing one item is $562, find the cost of producing 100 items.

63. The linear density of a rod of length 1 m is given by $\rho(x) = 1/\sqrt{x}$, in grams per centimeter, where x is measured in centimeters from one end of the rod. Find the mass of the rod.

64. A car is traveling at 50 mi/h when the brakes are fully applied, producing a constant deceleration of 22 ft/s^2. What is the distance traveled before the car comes to a stop?

65. What constant acceleration is required to increase the speed of a car from 30 mi/h to 50 mi/h in 5 s?

66. A car braked with a constant deceleration of 16 ft/s^2, producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

67. A car is traveling at 100 km/h when the driver sees an accident 80 m ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid a pileup?

68. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

69. Find the point on the hyperbola xy=8 that is closest to the point (3,0).

70. Find the smallest possible area of an isosceles triangle that is circumscribed about a circle of radius r.

71. Find the volume of the largest circular cone that can be inscribed in a sphere of radius r.

72. A metal storage tank with volume V is to be constructed in the shape of a right circular cylinder surmounted by a hemisphere. What dimensions will require the least amount of metal?

73. A hockey team plays in an arena with a seating capacity of 15,000 spectators. With the ticket price set at $12, average attendance at a game has been 11,000. A market survey indicates that for each dollar the ticket price is lowered, average attendance will increase by 1000. How should the owners of the team set the ticket price to maximize their revenue from ticket sales?

74. A manufacturer determines that the cost of making units of a commodity is $C(x)=1800+25x-0.2x^2+0.001x^3$ and the demand function is $p(x)=48.2-0.03x$.
(a) Graph the cost and revenue functions and use the graphs to estimate the production level for maximum profit.
(b) Use calculus to find the production level for maximum profit.

(c) Estimate the production level that minimizes the average cost.

75. A canister is dropped from a helicopter 500 m above the ground. Its parachute does not open, but the canister has been designed to withstand an impact velocity of 100 m/s. Will it burst?

76. In an automobile race along a straight road, car A passed car B twice. Prove that at some time during the race their accelerations were equal. State the assumptions that you make.

Chapter 5.

77. Oil leaked from a tank at a rate of r(t) liters per hour. The rate decreased as time passed and values of the rate at two hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

78. Evaluate the Riemann sum for $f(x)=3-\frac{1}{2}x$, $2\le x\le 14$, with six subintervals, taking the sample points to be left endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.

79. If f(x)=e^x-2, $0\le x\le 2$, find the Riemann sum with n=4 correct to six decimal places, taking the sample points to be midpoints. What does the Riemann sum represent? Illustrate with a diagram.

80. If oil leaks from a tank at a rate of r(t) gallons per minute at time t, what does $\int_0^{120} r(t)dt$ represent?

81. A honeybee population starts with 100 bees and increases at a rate $n'(t)$ of bees per week. What does $100 + \int_0^{15} n'(t)dt$ represent?

82. The linear density of a rod of length 4 m is given by $\rho(x)=9+2\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

83. Water flows from the bottom of a storage tank at a rate r9t)=200-4t of liters per minute, where $0\le t\le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

84. An oil storage tank ruptures at time t=0 and oil leaks from the tank at a rate of r(t)=100$e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour?

85. A particle moves along a line with velocity function v(t)=t^2 - t, where v is measured in meters per second. Find (a) the displacement and (b) the distance traveled by the particle during the time interval [0,5].

86. The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals as indicated in the figure. Use the Midpoint Rule to estimate the area of the pool.

87. A cross-section of an airplane wing is shown. Measurements of the height of the wing, in centimeters, at 20-centimeter intervals are 5.8, 20.3, 26.7, 29.0, 27.7, 27.3, 23.8, 20.5, 15.1,

8.7, and 2.8. Use the Midpoint Rule to estimate the area of the wing's cross-section.

88. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.
(a) Which car is ahead after one minute? Explain.
(b) What is the meaning of the area of the shaded region?
(c) Which car is ahead after two minutes? Explain.
(d) Estimate the time at which the cars are again side by side.

89. Find the area of the region bounded by the parabola $y=x^2$, the tangent line to this parabola at (1,1), and the x-axis.

90. Find the number b such that the line y=b divides the region bounded by the curves $y=x^2$ and y=4 into two regions with equal area.

91. Find the values of c such that the area of the region bounded by the parabolas $y=x^2 – c^2$ and $y=c^2 – x^2$ is 576.

92. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

93. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

94. Find the volume of the described solid. A right circular cone with height h and base radius r.

95. Find the volume of the described solid. A frustum of a right circular cone with height h, lower base radius R, and top radius r.

96. Find the volume of the described solid. A cap of a sphere with radius r and height h.

97. Find the volume of the described solid. A frustum of a pyramid with square base of side b, square top of side a, and height h.

98. Find the volume of the described solid. A pyramid with height h and rectangular base with dimensions b and 2b.

99. Find the volume of the described solid. A pyramid with height h and base an equilateral triangle with side a (a tetrahedron)

100. Find the volume (just the integral formula) of the inscribed solid. The base of S is the parabolic region $\{(x,y)| \, x^2 \leq y \leq 1\}$ Cross sections perpendicular to the y axis are equilateral triangles.

101. The base of S is a circular disk with radius r. Parallel cross sections perpendicular to the base are squares.

102. The base of S is an elliptical region with boundary curve $9x^2+4y^2 = 36$. Cross-sections perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base.

103. The base of S is the triangular region with vertices (0,0), (1,0), and (0,1). Cross-sections perpendicular to the y–axis are equilateral triangles.

104. The base of S is the region enclosed by the parabola $y=1-x^2$ and the x-axis. Cross-sections perpendicular to the x-axis are squares.

105. The base of is a circular disk with radius r. Parallel cross-sections perpendicular to the base are isosceles triangles with height h and unequal side in the base.
(a) Set up an integral for the volume of S.
(b) By interpreting the integral as an area, find the volume of S.

106. Find the volume common to two circular cylinders, each with radius r, if the axes of the cylinders intersect at right angles.

107. Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere.

108. A bowl is shaped like a hemisphere with diameter 30 cm. A ball with diameter 10 cm is placed in the bowl and water is poured into the bowl to a depth of h centimeters. Find the volume of water in the bowl.

109. A hole of radius r is bored through a cylinder of radius R>r at right angles to the axis of the cylinder. Set up, but do not evaluate, an integral for the volume cut out.

110. A hole of radius r is bored through the center of a sphere of radius R>r. Find the volume of the remaining portion of the sphere.

111. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis. Sketch the region and a typical shell.

112. Let V be the volume of the solid obtained by rotating about the y-axis the region bounded by $y = \sqrt{x}$ and $y=x^2$. Find V both by slicing and by cylindrical shells. In both cases draw a diagram to explain your method.

113. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis. Sketch the region and a typical shell.

114. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a typical shell.

115. How much work is done in lifting a 40-kg sandbag to a height of 1.5 m?

116. Find the work done if a constant force of 100 lb is used to pull a cart a distance of 200 ft.

117. A particle is moved along the x-axis by a force that measures $10/(1+x)^2$ pounds at a point x feet from the origin. Find the work done in moving the particle from the origin to a distance of 9 ft.

118. A force of 10 lb is required to hold a spring stretched 4 in. beyond its natural length. How much work is done in stretching it from its natural length to 6 in. beyond its natural length?

119. A spring has a natural length of 20 cm. If a 25-N force is required to keep it stretched to a length of 30 cm, how much work is required to stretch it from 20 cm to 25 cm?

120. Suppose that 2 J of work is needed to stretch a spring from its natural length of 30 cm to a length of 42 cm.
(a) How much work is needed to stretch the spring from 35 cm to 40 cm?
(b) How far beyond its natural length will a force of 30 N keep the spring stretched?

121. If the work required to stretch a spring 1 ft beyond its natural length is 12 ft-lb, how much work is needed to stretch it 9 in. beyond its natural length?

122. A spring has natural length 20 cm. Compare the work W_1 done in stretching the spring from 20 cm to 30 cm with the work W_2 done in stretching it from 30 cm to 40 cm. How are W_2 and W_1 related?

123. If 6 J of work is needed to stretch a spring from 10 cm to 12 cm and another 10 J is needed to stretch it from 12 cm to 14 cm, what is the natural length of the spring?

124. A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high.
(a) How much work is done in pulling the rope to the top of the building?
(b) How much work is done in pulling half the rope to the top of the building?

125. A chain lying on the ground is 10 m long and its mass is 80 kg. How much work is required to raise one end of the chain to a height of 6 m?

126. A cable that weighs 2 b/ft is used to lift 800 lb of coal up a mine shaft 500 ft deep. Find the work done.

127. A bucket that weighs 4 lb and a rope of negligible weight are used to draw water from a well that is 80 ft deep. The bucket is filled with 40 lb of water and is pulled up at a rate of 2 ft/s, but water leaks out of a hole in the bucket at a rate of 0.2 lb/s. Find the work done in pulling the bucket to the top of the well.

128. A leaky 10-kg bucket is lifted from the ground to a height of 12 m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12 m level. How much work is done?

129. A 10-ft chain weighs 25 lb and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end.

130. An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that the density of water is $1000 kg/m^3$.)

131. A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. How much work is required to pump all of the water out over the side? (Use the fact that water weighs 62.5 lb/ft^3.)

132. A tank is full of water. Find the work required to pump the water out of the spout. In Exercises 23 and 24 use the fact that water weighs 62.5 lb/ft^3.

133. Suppose that for the tank in Exercise 21 the pump breaks down after 4.7×10^5 J of work has been done. What is the depth of the water remaining in the tank?

134. (a) A cup of coffee has temperature 95 C and takes 30 minutes to cool to 61 C in a room with temperature 20 C. Use Newton's Law of Cooling (Section 3.8) to show that the temperature of the coffee after minutes is
$T(t) = 20 + 75e^{-kt}$
where k≈0.02.
(b) What is the average temperature of the coffee during the first half hour?

135. A particle that moves along a straight line has velocity $v(t)=t^2e^{-t}$ meters per second after t seconds. How far will it travel during the first t seconds?

136. Find the area of the crescent-shaped region (called a lune) bounded by arcs of circles with radii r and R. (See the figure.)

137. A water storage tank has the shape of a cylinder with diameter 10 ft. It is mounted so that the circular cross-sections are vertical. If the depth of the water is 7 ft, what percentage of the total capacity is being used?

138. Find the length of the arc of the curve from point P to point Q.

139. Find the arc length function for the curve $y = \sin^{-1} x + \sqrt{1-x^2}$ with starting point (0,1).

140. A steady wind blows a kite due west. The kite's height above ground from horizontal position x=0 to x=80 ft is given by $y = 150 - \frac{1}{40}(x-50)^2$. Find the distance traveled by the kite.

141. A hawk flying at 15 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation
$$y = 180 - \frac{x^2}{45}$$
until it hits the ground, where y is its height above the ground and x is the horizontal distance traveled in meters. Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground. Express your answer correct to the nearest tenth of a meter.

142. Find the area of the surface obtained by rotating the curve about the x-axis.
$y=x^3$, $0\le x\le 2$.

143. Use Simpson's Rule with n=10 to approximate the area of the surface obtained by rotating the curve about the x-axis. Compare your answer with the value of the integral produced by your calculator. $y=\ln x$, $1\le x\le 3$.

144. If the infinite curve $y=e^{-x}$, $x\ge 0$, is rotated about the x-axis, find the area of the resulting surface.

145. Find the area of the surface obtained by rotating the circle $x^2 + y^2 = r^2$ about the line y=r.

146. A large tank is designed with ends in the shape of the region between the curves $y=\frac{1}{2}x^2$ and y=12, measured in feet. Find the hydrostatic force on one end of the tank if it is filled to a depth of 8 ft with gasoline. (Assume the gasoline's density is 42.0 lb/ft^3.)

147. A trough is filled with a liquid of density 840 kg/m^3 . The ends of the trough are equilateral triangles with sides 8 m long and vertex at the bottom. Find the hydrostatic force on one end of the trough.

148. A vertical dam has a semicircular gate as shown in the figure. Find the hydrostatic force against the gate.

149. A cube with 20-cm-long sides is sitting on the bottom of an aquarium in which the water is one meter deep. Estimate the hydrostatic force on (a) the top of the cube and (b) one of the sides of the cube.

150. A dam is inclined at an angle 30° of from the vertical and has the shape of an isosceles trapezoid 100 ft wide at the top and 50 ft wide at the bottom and with a slant height of 70 ft. Find the hydrostatic force on the dam when it is full of water.

151. Find the centroid of the region bounded by the given curves. $y=x^2$, $x=y^2$.

152. The marginal cost function $C'(x)$ was defined to be the derivative of the cost function. (See Sections 3.7 and 4.7.) If the marginal cost of manufacturing x meters of a fabric is $C'(x)=5-0.008x+0.000009x^2$ (measured in dollars per meter) and the fixed start-up cost is C(0)=$20,000, use the Net Change Theorem to find the cost of producing the first 2000 units.

153. The marginal revenue from the sale of units of a product is 12-0.0004x. If the revenue from the sale of the first 1000 units is $12,400, find the revenue from the sale of the first 5000 units.

154. The marginal cost of producing units of a certain product is $74+1.1x-0.002x^2+0.00004x^3$ (in dollars per unit). Find the increase in cost if the production level is raised from 1200 units to 1600 units.

155. The demand function for a certain commodity is p=20-0.05x. Find the consumer surplus when the sales level is 300. Illustrate by drawing the demand curve and identifying the consumer surplus as an area.

156. A demand curve is given by p=450/(x+8). Find the consumer surplus when the selling price is $10

157. If a supply curve is modeled by the equation $p = 200 + 0.2x^{3/2}$, find the producer surplus when the selling price is \$400.

158. A movie theater has been charging \$7.50 per person and selling about 400 tickets on a typical weeknight. After surveying their customers, the theater estimates that for every 50 cents that they lower the price, the number of moviegoers will increase by 35 per night. Find the demand function and calculate the consumer surplus when the tickets are priced at \$6.00.

159. Suppose you have just poured a cup of freshly brewed coffee with temperature 95^{o}C in a room where the temperature is 20^{o}C.
(a) When do you think the coffee cools most quickly? What happens to the rate of cooling as time goes by? Explain.
(b) Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large. Write a differential equation that expresses Newton's Law of Cooling for this particular situation. What is the initial condition? In view of your answer to part (a), do you think this differential equation is an appropriate model for cooling?

160. Find an equation of the curve that passes through the point (0,1) and whose slope at (x,y) is xy.

161. A sphere with radius 1m has temperature 15^{o}C. It lies inside a concentric sphere with radius 2 m and temperature 25^{o}C. The temperature T(r) at a distance r from the common center of the spheres satisfies the differential equation

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0$$

If we let S=dT/dr, then satisfies a first-order differential equation. Solve it to find an expression for the temperature T(r) between the spheres.

162. A glucose solution is administered intravenously into the bloodstream at a constant rate r. As the glucose is added, it is converted into other substances and removed from the bloodstream at a rate that is proportional to the concentration at that time. Thus a model for the concentration C=C(t) of the glucose solution in the bloodstream is

$$\frac{dC}{dt} = r - kC$$

where k is a positive constant.
(a) Suppose that the concentration at time t=0 is C_o . Determine the concentration at any time by solving the differential equation.
(b) Assuming that $C_o<r/k$, find $\lim_{t\to\infty} C(t)$ and interpret your answer.

163. A tank contains 1000 L of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank (a) after t minutes and (b) after 20 minutes?

164. The air in a room with volume 180 m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of 2 m^3/min and the mixed air flows out at the same rate. Find the percentage of carbon dioxide in the room as a function of time. What happens in

the long run?

165. A vat with 500 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after an hour?

166. A tank contains 1000 L of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 5 L/min. Brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. How much salt is in the tank (a) after t minutes and (b) after one hour?

167. Suppose that a population develops according to the logistic equation

$$\frac{dP}{dt} = 0.05P - 0.0005P^2$$

where is measured in weeks.
(a) What is the carrying capacity? What is the value of k?
(b) A direction field for this equation is shown. Where are the slopes close to 0? Where are they largest? Which solutions are increasing? Which solutions are decreasing?

168. The population of the world was about 5.3 billion in 1990. Birth rates in the 1990s ranged from 35 to 40 million per year and death rates ranged from 15 to 20 million per year. Let's assume that the carrying capacity for world population is 100 billion.
(a) Write the logistic differential equation for these data. (Because the initial population is small compared to the carrying capacity, you can take k to be an estimate of the initial relative growth rate.)
(b) Use the logistic model to estimate the world population in the year 2000 and compare with the actual population of 6.1 billion.
(c) Use the logistic model to predict the world population in the years 2100 and 2500.
(d) What are your predictions if the carrying capacity is 50 billion?

169. One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the population who have heard the rumor and the fraction who have not heard the rumor.
(a) Write a differential equation that is satisfied by y.
(b) Solve the differential equation.
(c) A small town has 1000 inhabitants. At 8 AM, 80 people have heard a rumor. By noon half the town has heard it. At what time will 90% of the population have heard the rumor?

170. Biologists stocked a lake with 400 fish and estimated the carrying capacity (the maximal population for the fish of that species in that lake) to be 10,000. The number of fish tripled in the first year.
(a) Assuming that the size of the fish population satisfies the logistic equation, find an expression for the size of the population after t years.
(b) How long will it take for the population to increase to 5000?

171. In the circuit shown in Figure 4, a battery supplies a constant voltage of 40 V, the inductance is 2 H, the resistance is 10Ω, and I(0)=0.
(a) Find I(t).
(b) Find the current after 0.1 s.

172. Two new workers were hired for an assembly line. Jim processed 25 units during the first hour and 45 units during the second hour. Mark processed 35 units during the first hour and 50 units the second hour. Using the model of Exercise 31 and assuming that P(0)=0, estimate the maximum number of units per hour that each worker is capable of processing.

173. A tank with a capacity of 400 L is full of a mixture of water and chlorine with a concentration of 0.05 g of chlorine per liter. In order to reduce the concentration of chlorine, fresh water is pumped into the tank at a rate of 4 L/s. The mixture is kept stirred and is pumped out at a rate of 10 L/s. Find the amount of chlorine in the tank as a function of time.

174. (a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.
(b) Eliminate the parameter to find a Cartesian equation of the curve.
$x = 3t - 5$, $y = 2t + 1$

175. Match the graphs of the parametric equations x=f(t) and y=g(t) in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.

176. Match the parametric equations with the graphs labeled I-VI. Give reasons for your choices. (Do not use a graphing device.)

177. Find parametric equations for the path of a particle that moves along the circle $x^2 + (y - 1)^2 = 4$ in the manner described.
(a) Once around clockwise, starting at (2,1).
(b) Three times around counterclockwise, starting at (2,1).
(c) Halfway around counterclockwise, starting at (0,3).

178. If a and b are fixed numbers, find parametric equations for the curve that consists of all possible positions of the point P in the figure, using the angle as θ the parameter. Then eliminate the parameter and identify the curve.

179. Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.
$x = t^4 + 1$, $y = t^3 + t$, $t = -1$

180. Find an equation of the tangent to the curve at the given point by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter.
$x = 1 + \ln t$, $y = t^2 + 2$; (2,3)

181. Find the points on the curve where the tangent is horizontal or vertical. If you have a graphing device, graph the curve to check your work.
$x = 10 - t^2$, $y = t^3 - 12t$

182. Show that the curve x=cos t, y=sin t cos t has two tangents at (0,0) and find their equations. Sketch the curve.

183. Find equations of the tangents to the curve $x = 3t^2 + 1$, $y = 2t^3 + 1$ that pass through the point (4,3).

184. Find the area enclosed by the curve $x = t^2 - 2t$, $y = \sqrt{t}$ and the y-axis.

185. Find the area enclosed by the x-axis and the curve $x = 1 + e^t$, $y = t - t^2$.

186. Find the exact length of the curve.
$x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \le t \le 1$

187. Find the exact area of the surface obtained by rotating the given curve about the x-axis.
$x = t^3$, $y = t^2$, $0 \le t \le 1$

188. Find the surface area generated by rotating the given curve about the y-axis.
$x = 3t^2$, $y = 2t^3$, $0 \le t \le 5$

189. A string is wound around a circle and then unwound while being held taut. The curve traced by the point P at the end of the string is called the involute of the circle. If the circle has radius r and center O and the initial position of P is (r,0), and if the parameter θ is chosen as in the figure, show that parametric equations of the involute are
$x = r(\cos\theta + \theta\sin\theta)$ $x = r(\sin\theta - \theta\cos\theta)$

190. A cow is tied to a silo with radius r by a rope just long enough to reach the opposite side of the silo. Find the area available for grazing by the cow.

191. Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.
(a) $(1, \pi)$ (b) $(2, -2\pi/3)$ (c) $(-2, 3\pi/4)$

192. Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions.
$1 \le r \le 2$

193. Find the distance between the points with polar coordinates $(2, \pi/3)$ and $(4, 2\pi/3)$.

194. Find a formula for the distance between the points with polar coordinates (r_1, θ_1) and (r_2, θ_2).

195. Find a polar equation for the curve represented by the given Cartesian equation.
$x = 3$

196. Sketch the curve with the given polar equation.
$\theta = -\pi/6$

197. Match the polar equations with the graphs labeled I–VI. Give reasons for your choices. (Don't use a graphing device.)

198. Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.
$r = 2\sin\theta$, $\theta = \pi/6$.

199. Find the points on the given curve where the tangent line is horizontal or vertical.
$r = 3\cos\theta$

200. Find the area of the region that is bounded by the given curve and lies in the specified sector.
$r = \theta^2$

201. Find the area of the shaded region.

202. Sketch the curve and find the area that it encloses.
$r = 3\cos\theta$

203. Find the area of the region enclosed by one loop of the curve.
$r = \sin 2\theta$

204. Find the area of the region that lies inside the first curve and outside the second curve.
$r = 2\cos\theta$, r = 1

205. Find the exact length of the polar curve.
$r = 3\sin\theta,\ 0 \le \theta \le \pi/3$

206. Find the vertex, focus, and directrix of the parabola and sketch its graph.
$x = 2y^2$

207. Find the vertices and foci of the ellipse and sketch its graph.
$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

208. Find the vertices, foci, and asymptotes of the hyperbola and sketch its graph.
$$\frac{x^2}{144} - \frac{y^2}{25} = 1$$

209. Identify the type of conic section whose equation is given and find the vertices and foci.
$x^2 = y + 1$

210. Write a polar equation of a conic with the focus at the origin and the given data.
Parabola, eccentricity 7/4, directrix y = 6.

211. (a) Find the eccentricity, (b) identify the conic, (c) give an equation of the directrix, and (d) sketch the conic.
$$r = \frac{1}{1+\sin\theta}$$

212. Find an equation of the ellipse with foci (0,±4) and vertices (±5,0).

213. Find an equation of the parabola with focus (2,1) and directrix x = –4.

214. Find an equation of the hyperbola with foci (0, ±4) and asymptotes y = ±3x.

215. Find an equation of the ellipse with foci (3, ±2) and major axis with length 8.

216. Determine whether the sequence converges or diverges. If it converges, find the limit

$$a_n = 1-(0.2)^n$$

217. For what values of r is the sequence $\{nr^n\}$ convergent?

218. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

219. Find the values of x for which the series converges. Find the sum of the series for those values of x.

220. Use the Integral Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$$

221. Find the values of p for which the series is convergent.

222. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$ correct to three decimal places.

223. Use the sum of the first 10 terms to approximate the sum of the series. Estimate the error.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}}$$

224. Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$$

225. Approximate the sum of the series correct to four decimal places.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$$

226. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

227. Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$$

228. Find a power series representation for the function and determine the interval of convergence.

$$f(x)=\frac{1}{1+x}$$

229. Find a power series representation for the function and determine the radius of convergence.

$$f(x)=\ln(5-x)$$

230. Use a power series to approximate the definite integral to six decimal places.

$$\int_0^{0.2}\frac{1}{1+x^5}dx$$

231. Find the Taylor series for f(x) centered at the given value of a.

$f(x)=x^4-3x^2+1$, a=1

232. Use the binomial series to expand the function as a power series. State the radius of convergence.

$$\sqrt{1+x}$$

233. (a) Find the Taylor polynomials up to degree 6 for f(x)=cos x centered at a=0. Graph and these polynomials on a common screen.
(b) Evaluate and these polynomials at x=π/4, π/2, and π.
(c) Comment on how the Taylor polynomials converge to f(x).

234. A car is moving with speed 20 m/s and acceleration 2 m/s^2 at a given instant. Using a second-degree Taylor polynomial, estimate how far the car moves in the next second. Would it be reasonable to use this polynomial to estimate the distance traveled during the next minute?

235. Find the Taylor series of f(x)=sin x at a=π/6 .

236. Find an equation of the sphere that passes through the origin and whose center is (1, 2, 3).

237. A woman walks due west on the deck of a ship at 3 mi/h. The ship is moving north at a speed of 22 mi/h. Find the speed and direction of the woman relative to the surface of the water.

238. Ropes 3 m and 5 m in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes, fastened at different heights, make angles of 52^o and 40^o with the horizontal. Find the tension in each wire and the magnitude of each tension.

239. A clothesline is tied between two poles, 8 m apart. The line is quite taut and has negligible sag. When a wet shirt with a mass of 0.8 kg is hung at the middle of the line, the midpoint is pulled down 8 cm. Find the tension in each half of the clothesline.

240. The tension T at each end of the chain has magnitude 25 N. What is the weight of the chain?

241. Find the unit vectors that are parallel to the tangent line to the parabola $y=x^2$ at the point (2,4).

242. Find the angle between the vectors.

243. Determine whether the given vectors are orthogonal, parallel, or neither.

(a) $\vec{a} = \langle -5,3,7 \rangle$, $\vec{b} = \langle 6,-8,2 \rangle$

244. Find a unit vector that is orthogonal to both i + j and i + k.

245. Find the scalar and vector projections of b onto a.

246. Find the work done by a force $F = 8i - 6j + 9k$ that moves an object from the point (0,10,8) to the point (6,12,20) along a straight line. The distance is measured in meters and the force in newtons.

247. A tow truck drags a stalled car along a road. The chain makes an angle of 30^{o} with the road and the tension in the chain is 1500 N. How much work is done by the truck in pulling the car 1 km?

248. A sled is pulled along a level path through snow by a rope. A 30-lb force acting at an angle of 30^{o} above the horizontal moves the sled 80 ft. Find the work done by the force.

249. A boat sails south with the help of a wind blowing in the direction S36^{o}E with magnitude 400 lb. Find the work done by the wind as the boat moves 120 ft.

250. Find the angle between a diagonal of a cube and one of its edges.

251. Find the angle between a diagonal of a cube and a diagonal of one of its faces.

252. Find two unit vectors orthogonal to both $\langle 1,-1,1 \rangle$ and $\langle 0,4,4 \rangle$.

253. Use the scalar triple product to determine whether the points A(1,2,3), B(3, –1 ,6) C(5,2,0), and D(3, 6, –4) lie in the same plane.

254. A bicycle pedal is pushed by a foot with a 60-N force as shown. The shaft of the pedal is 18 cm long. Find the magnitude of the torque about P.

255. Find the magnitude of the torque about P if a 36-lb force is applied as shown.

256. A wrench 30 cm long lies along the positive -axis and grips a bolt at the origin. A force is applied in the direction $\langle 0,3,-4 \rangle$ at the end of the wrench. Find the magnitude of the force needed to supply 100 N·m of torque to the bolt.

257. The line through the origin and the point (1,2,3).

258. The line through the points (1,3,2) and (–4,3,0).

259. The plane that passes through the point (1,2,3) and contains the line x = 3t, y = 1 + t, z = 2 – t.

260. The plane that passes through the point (1,-1,1) and contains the line with symmetric equations x = 2y = 3z.

261. The plane that passes through the point $(-1,2,1)$and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

262. The plane that passes through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and is perpendicular to the plane $x + y - 2z = 1$.

263. Find an equation for the plane consisting of all points that are equidistant from the points (1,0,-2) and (3,4,0).

264. Find an equation of the plane with x-intercept a, y-intercept b, and z-intercept c.

265. Find parametric equations for the line through the point (0, 1,2) that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$.

266. Find parametric equations for the line through the point (0, 1, 2) that is perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$ and intersects this line.

267. Which of the following four planes are parallel? Are any of them identical?

P_1: $4x - 2y + 6z = 3$ P_2: $4x - 2y - 2z = 6$

P_3: $-6x + 3y - 9z = 5$ P_4: $z = 2x - y - 3$

268. Which of the following four lines are parallel? Are any of them identical?

L_1: $x = 1 + t$, $y = t$, $z = 2 - 5t$

L_2: $x + 1 = y - 2 = 1 - z$

L_3: $x = 1 + t$, $y = 4 + t$, $z = 1 - t$

L_4: $r = \langle 2,1,-3 \rangle + t\langle 2,2,-10 \rangle$

269. Find the distance from the point to the given plane.

(1, –2, 4), $3x + 2y + 6z = 5$

270. Find the distance between the given parallel planes.

$2x - 3y + z = 4$, $4x - 6y + 2z = 3$

271. Find equations of the planes that are parallel to the plane $x + 2y - 2z = 1$ and two units away from it.

272. Show that the lines with symmetric equations $x = y = z$ and $x + 1 = y/2 = z/3$ are skew, and find the distance between these lines.

273. (a) Find and identify the traces of the quadric surface $-x^2 - y^2 + z^2 = 1$ and explain why the graph looks like the graph of the hyperboloid of two sheets in Table 1.

(b) If the equation in part (a) is changed to $x^2 - y^2 + z^2$, what happens to the graph? Sketch the new graph.

274. Use traces to sketch and identify the surface

$x = y^2 + 4z^2$

275. Reduce the equation to one of the standard forms, classify the surface, and sketch it.
$z^2 = 4x^2 + 9y^2 + 36$

276. Sketch the region bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 = 1$ for $1 \le z \le 2$.

277. Sketch the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$

278. Find an equation for the surface obtained by rotating the parabola $y = x^2$ about the y-axis.

279. Find an equation for the surface obtained by rotating the line $x = 3y$ about the x-axis.

280. Find an equation for the surface consisting of all points that are equidistant from the point (–1,0,0) and the plane x = 1. Identify the surface.

281. Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane. Identify the surface.

282. A cooling tower for a nuclear reactor is to be constructed in the shape of a hyperboloid of one sheet (see the photo on page 810). The diameter at the base is 280 m and the minimum diameter, 500 m above the base, is 200 m. Find an equation for the tower.

283. (a) Find an equation of the sphere that passes through the point (6, –2, 3) and has center (–1,2,1).
(b) Find the curve in which this sphere intersects the yz-plane.
(c) Find the center and radius of the sphere
$x^2 + y^2 + z^2 - 8x + 2y + 6z + 1 = 0$

284. A constant force $F = 3i + 5j + 10k$ moves an object along the line segment from (1,0,2) to (5,3,8). Find the work done if the distance is measured in meters and the force in newtons.

285. A boat is pulled onto shore using two ropes, as shown in the diagram. If a force of 255 N is needed, find the magnitude of the force in each rope.

286. Find the point in which the line with parametric equations $x = 2 - t$, $y = 1 + 3t$, $z = 4t$ intersects the plane $2x - y + z = 2$.

287. Find the distance from the origin to the line $x = 1 + t$, $y = 2 - t$, $z = -1 + 2t$.

288. Determine whether the lines given by the symmetric equations

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and

$$\frac{x+1}{6} = \frac{y-3}{-1} = \frac{z+5}{2}$$

are parallel, skew, or intersecting.

289. (a) Show that the planes $x + y - z = 1$ and $2x - 3y + 4z = 5$ are neither parallel nor perpendicular.
(b) Find, correct to the nearest degree, the angle between these planes.

290. Find an equation of the plane through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and perpendicular to the plane $x + y - 2z = 1$.

291. (a) Find an equation of the plane that passes through the points A(2,1,1), B(–1, –1,10) and C(1,3, –4).
(b) Find symmetric equations for the line through B that is perpendicular to the plane in part (a).
(c) A second plane passes through (2,0,4) and has normal vector $\langle 2,-4,-3\rangle$. Show that the acute angle between the planes is approximately 43^{o}.
(d) Find parametric equations for the line of intersection of the two planes.

292. Find the distance between the planes $3x + y - 4z = 2$ and $3x + y - 4z = 24$.

293. An ellipsoid is created by rotating the ellipse $4x^2 + y^2 = 16$ about the x-axis. Find an equation of the ellipsoid.

294. A surface consists of all points such that the distance from P to the plane $y = 1$ is twice the distance from P to the point (0, –1,0). Find an equation for this surface and identify it.

295. Find the domain of the vector function
$r(t) = \left\langle \sqrt{4-t^2}, e^{-3t}, \ln(t+1) \right\rangle$

296. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which increases: $r(t) = \langle \sin t, t \rangle$

297. Find a vector equation and parametric equations for the line segment that joins P to Q
P(0,0,0), Q (1,2,3)

298. Show that the curve with parametric equations $x = \sin t$, $y = \cos t$, $z = \sin^2 t$ lies on the cone $z^2 = x^2 + y^2$, and use this fact to help sketch the curve.

299. At what points does the curve $r(t) = ti + (2t - t^2)k$ intersect the paraboloid $z = x^2 + y^2$?

300. At what points does the helix $r(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

301. Find a vector function that represents the curve of intersection of the two surfaces.
The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

302. Two particles travel along the space curves
$r_1(t) = \langle t, t^2, t^3 \rangle \qquad r_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle$
Do the particles collide? Do their paths intersect?

303. Find the derivative of the vector function $r(t) = \langle t\sin t, t^2, t\cos 2t \rangle$

304. Find the unit tangent vector T(t) at the point with the given value of the parameter t.
$r(t) = \langle te^{-t}, 2\arctan t, 2e^{t} \rangle$, t = 0

305. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.
$x = 1 + 2\sqrt{t}$ $y = t^3 - t$ $z = t^3 + t$; (3,0,2)

306. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point. Illustrate by graphing both the curve and the tangent line on a common screen.
$x = t$ $y = e^{-t}$ $z = 2t - t^2$; (0,1,0)

307. The curves $r_1(t) = \langle t, t^2, t^3 \rangle$ and $r_2(t) = \langle \sin t, \sin 2t, t \rangle$ intersect at the origin. Find their angle of intersection correct to the nearest degree.

308. At what point do the curves $r_1(t) = \langle t, 1 - t, 3 + t^2 \rangle$ and $r_2(s) = \langle 3 - s, s - 2, s^2 \rangle$ intersect? Find their angle of intersection correct to the nearest degree.

309. If a curve has the property that the position vector r(t) is always perpendicular to the tangent vector r ′(t), show that the curve lies on a sphere with center the origin.

310. Find the length of the curve. $r(t) = \langle 2\sin t, 5t, 2\cos t \rangle$, $-10 \le t \le 1$.

311. Graph the curve with parametric equations x = sin t, y = sin 2t, z = sin 3t. Find the total length of this curve correct to four decimal places.

312. Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface 3z = xy. Find the exact length of C from the origin to the point (6, 18, 36).

313. Find, correct to four decimal places, the length of the curve of intersection of the cylinder $4x^2 + y^2 = 4$ and the plane x + y + z = 2.

314. Reparametrize the curve with respect to arc length measured from the point where t = 0 in the direction of increasing t.
r(t) = 2ti + (1 – 3t)j + (5 +4t)k

315. Suppose you start at the point (0,0,3) and move 5 units along the curve x = 3sin t, y = 4t, z = 3cos t in the positive direction. Where are you now?

316. Find the curvature of $r(t) = \langle e^t \cos t, e^t \sin t, t \rangle$ at the point (1, 1, 1).

317. Find the vectors T, N, and B at the given point.
$r(t) = \left\langle t^2, \frac{2}{3}t^3, t \right\rangle$, $\left(1, \frac{2}{3}, 1\right)$

318. Find equations of the normal plane and osculating plane of the curve at the given point.
$x = 2\sin 3t$, $y = t$, $z = 2\cos 3t$; $(0, \pi, -2)$.

319. Find equations of the osculating circles of the ellipse $9x^2 + 4y^2 = 36$ at the points (2,0) and (0, 3). Use a graphing calculator or computer to graph the ellipse and both osculating circles on the same screen.

320. Find equations of the osculating circles of the parabola $y = \frac{1}{2}x^2$ at the points (0, 0) and (1, ½). Graph both osculating circles and the parabola on the same screen.

321. At what point on the curve $x = t^3$, $y = 3t$, $z = t^4$ is the normal plane parallel to the plane $6x + 6y - 8z = 1$?

322. Show that the curvature k is related to the tangent and normal vectors by the equation $\frac{dT}{ds} = kN$.

323. Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t.
$r(t) = \left\langle -\frac{1}{2}t^2, t \right\rangle$, $t = 2$

324. Find the velocity, acceleration, and speed of a particle with the given position function.
$r(t) = \langle t^2 + 1, t^3, t^2 - 1 \rangle$

325. Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position.
$a(t) = i + 2j$, $v(0) = k$, $r(0) = i$

326. The position function of a particle is given by $r(t) = \langle t^2, 5t, t^2 - 16t \rangle$. When is the speed a minimum?

327. What force is required so that a particle of mass has the position function $r(t) = t^3i + t^2j + t^3k$

328. A force with magnitude 20 N acts directly upward from the xy-plane on an object with mass 4 kg. The object starts at the origin with initial velocity $v(0) = i - j$. Find its position function and its speed at time t.

329. Show that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.

330. A projectile is fired with an initial speed of 500 m/s and angle of elevation 30^o. Find (a) the range of the projectile, (b) the maximum height reached, and (c) the speed at impact.

331. A ball is thrown at an angle of 45^o to the ground. If the ball lands 90 m away, what was the initial speed of the ball?

332. A gun is fired with angle of elevation 30^o. What is the muzzle speed if the maximum height of the shell is 500 m?

333. A gun has muzzle speed 150 m/s. Find two angles of elevation that can be used to hit a target 800 m away.

334. A batter hits a baseball 3 ft above the ground toward the center field fence, which is 10 ft high and 400 ft from home plate. The ball leaves the bat with speed 115 ft/s at an angle 50^o above the horizontal. Is it a home run? (In other words, does the ball clear the fence?)

335. A medieval city has the shape of a square and is protected by walls with length 500 m and height 15 m. You are the commander of an attacking army and the closest you can get to the wall is 100 m. Your plan is to set fire to the city by catapulting heated rocks over the wall (with an initial speed of 80 m/s). At what range of angles should you tell your men to set the catapult? (Assume the path of the rocks is perpendicular to the wall.)

336. A ball with mass 0.8 kg is thrown southward into the air with a speed of 30 m/s at an angle of 30^o to the ground. A west wind applies a steady force of 4 N to the ball in an easterly direction. Where does the ball land and with what speed?

337. Water traveling along a straight portion of a river normally flows fastest in the middle, and the speed slows to almost zero at the banks. Consider a long straight stretch of river flowing north, with parallel banks 40 m apart. If the maximum water speed is 3 m/s, we can use a quadratic function as a basic model for the rate of water flow units from the west bank: $f(x) = \frac{3}{400}x(40 - x)$.

(a) A boat proceeds at a constant speed of 5 m/s from a point A on the west bank while maintaining a heading perpendicular to the bank. How far down the river on the opposite bank will the boat touch shore? Graph the path of the boat.

(b) Suppose we would like to pilot the boat to land at the point B on the east bank directly opposite A. If we maintain a constant speed of and a constant heading, find the angle at which the boat should head. Then graph the actual path the boat follows. Does the path seem realistic?

338. Another reasonable model for the water speed of the river in Exercise 31 is a sine function: $f(x) = 3\sin(\pi x/40)$. If a boater would like to cross the river from A to B with constant heading and a constant speed of 5 m/s, determine the angle at which the boat should head.

339. Find the tangential and normal components of the acceleration vector.

$r(t) = (3t - t^3)i + 3t^2j$

Chapter 14

340. Match the function with its graph (labeled I–VI).Give reasons for your choices.

$f(x,y) = |x| + |y|$

341. A contour map for a function f is shown. Use it to estimate the values of f(–3,3) and f(3, –2). What can you say about the shape of the graph?

342. Two contour maps are shown. One is for a function f whose graph is a cone. The other is for a function g whose graph is a paraboloid. Which is which, and why?

343. Draw a contour map of the function showing several level curves.
$f(x,y) = (y - 2x)^2$

344. Describe the level surfaces of the function
$f(x,y,z) = x + 3y + 5z$

345. Find the limit, if it exists, or show that the limit does not exist.
$$\lim_{(x,y)\to(1,2)} (5x^3 - x^2y^2)$$

346. Determine the set of points at which the function is continuous.
$$F(x,y) = \frac{\sin(xy)}{e^x - y^2}$$

347. Use polar coordinates to find the limit. [If (r, θ) are polar coordinates of the point (x,y) with $r \geq 0$, note that $r \to 0^+$ as $(x,y) \to (0,0)$.]

348. The following surfaces, labeled a, b, and c, are graphs of a function f and its partial derivatives f_x and f_y. Identify each surface and give reasons for your choices.

349. A contour map is given for a function f. Use it to estimate $f_x(2,1)$ and $f_y(2,1)$.

350. If $f(x,y) = 16 - 4x^2 - y^2$, find $f_x(1,2)$ and $f_y(1,2)$ and interpret these numbers as slopes. Illustrate with either hand-drawn sketches or computer plots.

351. Find the first partial derivatives of the function
$f(x,y) = y^5 - 3xy$

352. Use the definition of partial derivatives as limits (4) to find $f_x(x,y)$ and $f_y(x,y)$.
$f(x,y) = xy^2 - x^3y$

353. Find all the second partial derivatives
$f(x,y) = x^3y^5 + 2x^4y$

354. Find the indicated partial derivative.

355. Use the table of values of f(x,y) to estimate the values of $f_x(3,2)$, $f_x(3,2.2)$, and $f_{xy}(3,2)$.

356. Level curves are shown for a function . Determine whether the following partial derivatives are positive or negative at the point P.

357. The total resistance R produced by three conductors with resistances R_1, R_2, R_3 connected in a parallel electrical circuit is given by the formula
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Find $\frac{\partial R}{\partial R_1}$.

358. You are told that there is a function f whose partial derivatives are $f_x(x,y) = x + 4y$ and $f_y(x,y) = 3x - y$. Should you believe it?

359. The paraboloid $z = 6 - x - x^2 - 2y^2$ intersects the plane x = 1 in a parabola. Find parametric equations for the tangent line to this parabola at the point (1,2, –4). Use a computer to graph the paraboloid, the parabola, and the tangent line on the same screen.

360. The ellipsoid $4x^2 + 2y^2 + z^2 = 16$ intersects the plane y = 2 in an ellipse. Find parametric equations for the tangent line to this ellipse at the point (1,2,2).

361. Find an equation of the tangent plane to the given surface at the specified point.
$z = 4x^2 - y^2 + 2y$, (–1, 2,4)

362. Explain why the function is differentiable at the given point. Then find the linearization of the function at that point.
$f(x,y) = x\sqrt{y}$, (1,4)

363. Find the linear approximation of the function $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ at (2,1) and use it to approximate f(1.95,1.08).

364. The wave heights h in the open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function h=f(v,t) are recorded in feet in the following table.
Use the table to find a linear approximation to the wave height function when v is near 40 knots and t is near 20 hours. Then estimate the wave heights when the wind has been blowing for 24 hours at 43 knots.

365. Use the table in Example 3 to find a linear approximation to the heat index function when the temperature is near 94°F and the relative humidity is near 80%. Then estimate the heat index when the temperature is 95°F and the relative humidity is 78%.

366. The wind-chill index W is the perceived temperature when the actual temperature is T and the wind speed is v, so we can write W=f(T,v). The following table of values is an excerpt from Table 1 in Section 14.1. Use the table to find a linear approximation to the wind-chill index function when is near –15°C and v is near 50 km/h. R Then estimate the wind-chill index when the temperature is –17°C and the wind speed is 55 km/h.

367. Find the differential of the function.
$v = \cos xy$

368. If $z = 5x^2 + y^2$ and (x,y) changes from (1,2) to (1.05,2.1), compare the values of Δz and dz.

369. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

370. The dimensions of a closed rectangular box are measured as 80 cm, 60 cm, and 50 cm, respectively, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box.

371. Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

372. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

373. A boundary stripe 3 in. wide is painted around a rectangle whose dimensions are 100 ft by 200 ft. Use differentials to approximate the number of square feet of paint in the stripe.

374. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation PV=8.31T, where is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12 L to 12.3 L and the temperature decreases from 310 K to 305 K.

375. If R is the total resistance of three resistors, connected in parallel, with resistances R_1, R_2, R_3 then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If the resistances are measured in ohms as $R_1 = 25\ \Omega$, $R_2 = 40\ \Omega$, and $R_3 = 50\ \Omega$, with a possible error of 0.5% in each case, estimate the maximum error in the calculated value of R.

376. Four positive numbers, each less than 50, are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from the rounding.

377. A model for the surface area of a human body is given by $S= 0.1091w^{0.425}h^{0.725}$, where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet. If the errors in measurement of w and h are at most 2%, use differentials to estimate the maximum percentage error in the calculated surface area.

378. Suppose you need to know an equation of the tangent plane to a surface S at the point P(2,1,3). You don't have an equation for S but you know that the curves

$$r_1(t) = \left\langle 2+3t, 1-t^2, 3-4t+t^2 \right\rangle$$

$$r_2(u) = \left\langle 1+u^2, 2u^3-1, 2u+1 \right\rangle$$

both lie on S. Find an equation of the tangent plane at P.

379. Show that the function is differentiable by finding values of ε_1 and ε_2 that satisfy Definition 7.

$f(x,y) = x^2 + y^2$

380. Use the Chain Rule to find dz/dt or dw/dt.
$z = x^2 + y^2 + xy$, $x = \sin t$, $y = e^t$

381. Use the Chain Rule to find the indicated partial derivatives
$z = x^2 + xy^3$, $x = uv^2 + w^3$, $y = u + ve^w$

382. The temperature at a point (x,y) is T(x,y), measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1+t}$, $y = 2 + \frac{1}{3}t$ where x and y are measured in centimeters. The temperature function satisfies $T_x(2,3) = 4$ and $T_y(2,3) = 4$. How fast is the temperature rising on the bug's path after 3 seconds?

383. Wheat production W in a given year depends on the average temperature T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15°C/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that, at current production levels, $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = 8$.
(a) What is the significance of the signs of these partial derivatives?
(b) Estimate the current rate of change of wheat production, dW/dt.

384. The speed of sound traveling through ocean water with salinity 35 parts per thousand has been modeled by the equation
$C = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + 0.016D$
where C is the speed of sound (in meters per second), T is the temperature (in degrees Celsius), and D is the depth below the ocean surface (in meters). A scuba diver began a leisurely dive into the ocean water; the diver's depth and the surrounding water temperature over time are recorded in the following graphs. Estimate the rate of change (with respect to time) of the speed of sound through the ocean water experienced by the diver 20 minutes into the dive. What are the units?

385. The radius of a right circular cone is increasing at a rate of in□s while its height is decreasing at a rate of in□s. At what rate is the volume of the cone changing when the radius is 120 in. and the height is 140 in.?

386. The length l, width w, and height h of a box change with time. At a certain instant the dimensions are l = 1 m and w = h = 2 m, and l and w are increasing at a rate of 2 m/s while is decreasing at a rate of 3 m/s. At that instant find the rates at which the following quantities are changing.
(a) The volume
(b) The surface area
(c) The length of a diagonal

387. The voltage V in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance R is slowly increasing as the resistor heats up. Use Ohm's Law, V=IR, to find how the current is changing at the moment when R=400 Ω, I=0.08 A, dV/dt =–0.01 V/s, and dR/dt = 0.03 Ω/s.

388. The pressure of 1 mole of an ideal gas is increasing at a rate of 0.05 kPa/s and the temperature is increasing at a rate of 0.15 K/s. Use the equation in Example 2 to find the rate of change of the volume when the pressure is 20 kPa and the temperature is 320 K.

389. Car A is traveling north on Highway 16 and car B is traveling west on Highway 83. Each car is approaching the intersection of these highways. At a certain moment, car A is 0.3 km from the intersection and traveling at 90 km/h while car B is 0.4 km from the intersection and traveling at 80 km/h. How fast is the distance between the cars changing at that moment?

390. One side of a triangle is increasing at a rate of and a second side is decreasing at a rate of 2 cm/s. If the area of the triangle remains constant, at what rate does the angle between the sides change when the first side is 20 cm long, the second side is 30 cm, and the angle is $\pi/6$?

391. Level curves for barometric pressure (in millibars) are shown for 6:00 AM on November 10, 1998. A deep low with pressure 972 mb is moving over northeast Iowa. The distance along the red line from K (Kearney, Nebraska) to S (Sioux City, Iowa) is 300 km. Estimate the value of the directional derivative of the pressure function at Kearney in the direction of Sioux City. What are the units of the directional derivative?

392. The contour map shows the average maximum temperature for November 2004 (in ^{o}C). Estimate the value of the directional derivative of this temperature function at Dubbo, New South Wales, in the direction of Sydney. What are the units?

393. Find the directional derivative of f at the given point in the direction indicated by the angle θ.
$f(x,y)= x^2y^3 - y^4$, (2,1), $\theta = \pi/4$

394. Find the directional derivative of the function at the given point in the direction of the vector v.
$f(x,y) = \ln(x^2 + y^2)$, (2,1), $v = \langle -1,2 \rangle$

395. Find the directional derivative of $f(x,y,z) = xy + yz + zx$ at P(1,-1,3) in the direction of Q(2,4,5).

396. Find the maximum rate of change of f at the given point and the direction in which it occurs.
$f(x,y) = y^2/x$, (2,4)

397. (a) Show that a differentiable function decreases most *f* rapidly at in the direction opposite to the gradient vector, that is, in the direction of $-\nabla f(x)$.
(b) Use the result of part (a) to find the direction in which the function $f(x,y) = x^4y - x^2y^3$ decreases fastest at the point (2, –3).

398. Find the directions in which the directional derivative of $f(x,y) = ye^{-xy}$ at the point (0,2) has the value 1.

399. Find all points at which the direction of fastest change of the function $f(x,y) = x^2 + y^2 - 2x - 4y$ is $i + j$.

400. Near a buoy, the depth of a lake at the point with coordinates (,y) is $z = 200 + 0.02x^2 - 0.001y^3$, where x, y and z are measured in meters. A fisherman in a small boat starts at the point (80, 60) and moves toward the buoy, which is located at (0,0). Is the water under the boat getting deeper or shallower when he departs? Explain.

401. The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point (1,2,2) is 120^{o}.
(a) Find the rate of change of T at (1,2,2) in the direction toward the point (2,1,3).

(b) Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points toward the origin.

402. Suppose that over a certain region of space the electrical potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$.
(a) Find the rate of change of the potential at P(3,4,5) in the direction of the vector v = i + j – k.
(b) In which direction does V change most rapidly at P?
(c) What is the maximum rate of change at P?

403. Suppose you are climbing a hill whose shape is given by the equation $z = 1000 - 0.005x^2 - 0.01y^2$, where x, y, and z are measured in meters, and you are standing at a point with coordinates (60,40,966).
The positive x-axis points east and the positive y-axis points north.
(a) If you walk due south, will you start to ascend or descend? At what rate?
(b) If you walk northwest, will you start to ascend or descend? At what rate?
(c) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?

404. Let f be a function of two variables that has continuous partial derivatives and consider the points A(1,3), B(3,3), C(1,7), and D(6,15). The directional derivative of f at A in the direction of the vector $\overrightarrow{AB}$ is 3 and the directional derivative at A in the direction of $\overrightarrow{AC}$ is 26. Find the directional derivative of at in the direction of the vector $\overrightarrow{AD}$.

405. Find equations of (a) the tangent plane and (b) the normal line to the given surface at the specified point.
$2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10$, (3,3,5)

406. If f(x,y) = xy, find the gradient vector $\nabla f(3,2)$ and use it to find the tangent line to the level curve f(x,y) = 6 at the point (3,2). Sketch the level curve, the tangent line, and the gradient vector.

407. Find the equation of the tangent plane to the hyperboloid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ at the point (x_o, y_o, z_o) and express it in a form similar to the one in Exercise 49.

408. At what point on the paraboloid $y = x^2 + z^2$ is the tangent plane parallel to the plane x + 2y + 3z = 1?

409. Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane z = x + y?

410. Show that every plane that is tangent to the cone $x^2 + y^2 = z^2$ passes through the origin.

411. Show that every normal line to the sphere $x^2 + y^2 + z^2 = 1$ passes through the center of the sphere.

412. Show that the sum of the x-, y-, and z-intercepts of any tangent plane to the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{c}$ is a constant.

413. Show that the pyramids cut off from the first octant by any tangent planes to the surface $xyz = 1$ at points in the first octant must all have the same volume.

414. (a) The plane $y + z = 3$ intersects the cylinder $x^2 + y^2 = 5$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point (1,2,1).
(b) Graph the cylinder, the plane, and the tangent line on the same screen.

415. Find the local maximum and minimum values and saddle point(s) of the function. If you have three-dimensional graphing software, graph the function with a domain and viewpoint that reveal all the important aspects of the function.
$f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

416. Show that $f(x,y) = x^2 + 4y^2 - 4xy + 2$ has an infinite number of critical points and that $D = 0$ at each one. Then show that f has a local (and absolute) minimum at each critical point.

417. Find the absolute maximum and minimum values of f on the set D.
$f(x,y) = 1 + 4x - 5y$, D is the closed triangular region with vertices (0,0), (2,0), and (3,0).

418. Find the shortest distance from the point (2,1,– 1) to the plane $x + y - z = 1$.

419. Find the point on the plane $x - y + z = 4$ that is closest to the point (1,2,3).

420. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point (4, 2, 0).

421. Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin.

422. Find three positive numbers whose sum is 100 and whose product is a maximum.

423. Find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible.

424. Find the maximum volume of a rectangular box that is inscribed in a sphere of radius r.

425. Find the dimensions of the box with volume 1000 cm^3 that has minimal surface area.

426. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.

427. Find the dimensions of the rectangular box with largest volume if the total surface area is given as 64 cm^2.

428. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant c.

429. The base of an aquarium with given volume V is made of slate and the sides are made of glass. If slate costs five times as much (per unit area) as glass, find the dimensions of the aquarium that minimize the cost of the materials.

430. A cardboard box without a lid is to have a volume of 32000 cm^3. Find the dimensions that minimize the amount of cardboard used.

431. A rectangular building is being designed to minimize heat loss. The east and west walls lose heat at a rate of 10 units/m^2 per day, the north and south walls at a rate of 8 units/m^2 per day, the floor at a rate of 1 units/m^2 per day, and the roof at a rate of 5 units/m^2 per day. Each wall must be at least 30 m long, the height must be at least 4 m, and the volume must be exactly 4000 m^3.
(a) Find and sketch the domain of the heat loss as a function of the lengths of the sides.
(b) Find the dimensions that minimize heat loss. (Check both the critical points and the points on the boundary of the domain.)
(c) Could you design a building with even less heat loss if the restrictions on the lengths of the walls were removed?

432. If the length of the diagonal of a rectangular box must be L, what is the largest possible volume?

433. Find an equation of the plane that passes through the point (1, 2, 3) and cuts off the smallest volume in the first octant.

434. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint(s).
$f(x,y) = x^2 + y^2$; $xy = 1$

435. Find the extreme values of f on the region described by the inequality.
$f(x,y) = 2x^2 + 3y^2 - 4x - 5$, $x^2 + y^2 \le 16$

436. The total production P of a certain product depends on the amount of labor L used and the amount K of capital investment. In Sections 14.1 and 14.3 we discussed how the Cobb-Douglas model $P = bL^{\alpha}K^{1-\alpha}$ follows from certain economic assumptions, where b and α are positive constants and $\alpha < 1$. If the cost of a unit of labor is m and the cost of a unit of capital is n, and the company can spend only dollars p as its total budget, then maximizing the production is subject to the constraint $mL + nK = p$. Show that the maximum production occurs when
$L = \frac{\alpha p}{m}$ and $k = \frac{(1-\alpha)p}{n}$

437. Use Lagrange multipliers to prove that the rectangle with maximum area that has a given perimeter p is a square.

438. Use Lagrange multipliers to prove that the triangle with maximum area that has a given perimeter p is equilateral.

439. Find the maximum and minimum volumes of a rectangular box whose surface area is 1500 cm^2 and whose total edge length is 200 cm.

440. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.

441. The plane $4x - 3y + 8z = 5$ intersects the cone in an ellipse.
(a) Graph the cone, the plane, and the ellipse.

(b) Use Lagrange multipliers to find the highest and lowest points on the ellipse.

442. Find equations of (a) the tangent plane and (b) the normal line to the given surface at the specified point.
$z = 3x^2 - y^2 + 2x$. (1, –2,1)

443. Find the points on the hyperboloid $x^2 + y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$.

444. The two legs of a right triangle are measured as 5 m and 12 m with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the maximum error in the calculated value of (a) the area of the triangle and (b) the length of the hypotenuse.

445. The length x of a side of a triangle is increasing at a rate of 3 in/s, the length y of another side is decreasing at a rate of 2 in/s, and the contained angle θ is increasing at a rate 0.05 of radian/s. How fast is the area of the triangle changing when x=40 in, y=50 in, and θ=π/6?

446. Find the gradient of the function $f(x,y,z)=z^2 e^{x\sqrt{y}}$.

447. (a) When is the directional derivative of f a maximum?
(b) When is it a minimum?
(c) When is it 0?
(d) When is it half of its maximum value?

448. Find the directional derivative of f at the given point in the indicated direction.
$f(x,y) = 2\sqrt{x} - y^2$, (1,5)
in the direction toward the point (4,1).

449. Find the maximum rate of change of $f(x,y)=x^2y + \sqrt{x}$ at the point (2,1). In which direction does it occur?

450. Find the direction in which $f(x,y,z)=ze^{xy}$ increases most rapidly at the point (0,1,2). What is the maximum rate of increase?

451. The contour map shows wind speed in knots during Hurricane Andrew on August 24, 1992. Use it to estimate the value of the directional derivative of the wind speed at Homestead, Florida, in the direction of the eye of the hurricane.

452. Find parametric equations of the tangent line at the point (–2,2,4) to the curve of intersection of the surface $z = 2x^2 - y^2$ and the plane z=4.

453. Find the local maximum and minimum values and saddle points of the function. If you have three-dimensional graphing software, graph the function with a domain and viewpoint that reveal all the important aspects of the function.
$f(x,y) = x^2 - xy + y^2 + 9x - 6y + 10$

454. Find the absolute maximum and minimum values of on the set D.
$f(x,y) = 4xy^2 – x^2y^2 – xy^3$; D is the closed triangulat region in the xy-plane with vertices (0,0),(0,6), and (6,0).

455. Use Lagrange multipliers to find the maximum and minimum values of subject to the given constraint(s).
$f(x,y) = x^2y$; $x^2 + y^2 = 1$

456. Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.

457. A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth (the perimeter of a cross-section perpendicular to the length) is at most 108 in. Find the dimensions of the package with largest volume that can be mailed.

458. A pentagon is formed by placing an isosceles triangle on a rectangle, as shown in the figure. If the pentagon has fixed perimeter , find the lengths of the sides of the pentagon that maximize the area of the pentagon.

459. A particle of mass m moves on the surface z=f(x,y). Let x=x(t) and y=y(t) be the x- and y-coordinates of the particle at time t.
(a) Find the velocity vector v and the kinetic energy $K = \frac{1}{2}m|v|^2$ of the particle.
(b) Determine the acceleration vector *a*.
(c)Let $z = x^2 + y^2$ and x(t) = t cos(t), y(t) = t sin t. Find the velocity vector, the kinetic energy, and the acceleration vector.

460. (a) Estimate the volume of the solid that lies below the surface z = xy and above the rectangle
$R = \{(x,y) \mid 0 \le x \le 6, 0 \le y \le 4\}$
Use a Riemann sum with m = 3, n = 2, and take the sample point to be the upper right corner of each square.
(b) Use the Midpoint Rule to estimate the volume of the solid in part (a).

461. If R = [–1,3] × [0,2] use a Riemann sum with m=4, n=2 to estimate the value of $\iint_R (y^2 - 2x^2)dA$.
Take the sample points to be the upper left corners of the squares.

462. (a) Use a Riemann sum with m=n=2 to estimate the value of $\iint_R \sin(x+y)dA$, where
R = [0,π] × [0,π]. Take the sample points to be lower left corners.
(b) Use the Midpoint Rule to estimate the integral in part (a).

463. (a) Estimate the volume of the solid that lies below the surface $z = x + 2y^2$ and above the rectangle R = [0,2] × [0,4]. Use a Riemann sum with m=n=2 and choose the sample points to be lower right corners.
(b) Use the Midpoint Rule to estimate the volume in part (a).

464. A table of values is given for a function f(x,y) defined on R = [1,3] × [0,4].
(a) Estimate $\iint_R f(x,y)dA$ using the Midpoint Rule with m = n = 2.

(b) Estimate the double integral with by choosing the sample points to be the points farthest from the origin.

465. A 20-ft-by-30-ft swimming pool is filled with water. The depth is measured at 5-ft intervals, starting at one corner of the pool, and the values are recorded in the table. Estimate the volume of water in the pool.

466. Let be the volume of the solid that lies under the graph of $f(x,y)=\sqrt{52-x^2-y^2}$ and above the rectangle given by $2 \le x \le 4$, $2 \le y \le 6$. We use the lines x = 3 and y = 4 to divide R into subrectangles. Let L and U be the Riemann sums computed using lower left corners and upper right corners, respectively. Without calculating the numbers V, L, and U, arrange them in increasing order and explain your reasoning.

467. The figure shows level curves of a function f in the square R = [0,2] × [0,2] . Use the Midpoint Rule with m=n=2 to estimate $\iint_R f(x,y)dA$. How could you improve your estimate?

468. A contour map is shown for a function on the square R = [0,4] × [0,4].

(a) Use the Midpoint Rule with m=n=2 to estimate the value of $\iint_R f(x,y)dA$.

(b) Estimate the average value of f.

469. The contour map shows the temperature, in degrees Fahrenheit, at 4:00 PM on February 26, 2007, in Colorado. (The state measures 388 mi east to west and 276 mi north to south.) Use the Midpoint Rule with m=n=4 to estimate the average temperature in Colorado at that time.

470. Evaluate the double integral by first identifying it as the volume of a solid.

$\iint_R 3dA$, R = {(x,y) | $-2 \le x \le 2$, $1 \le y \le 6$}

471. Find the volume of the solid that lies under the plane 3x + 2y + z = 12 and above the rectangle R = {(x,y) | $0 \le x \le 1$, $-2 \le y \le 3$}

472. Find the volume of the solid that lies under the hyperbolic paraboloid $z = 4 + x^2 - y^2$ and above the square R = [–1,1] × [0,2].

473. Find the volume of the solid lying under the elliptic paraboloid $x^2/4 + y^2/9 + z = 1$ and above the rectangle R = [–1,1] × [–2,2].

474. Find the volume of the solid enclosed by the surface $z = 1 + e^x$ and the planes $x = \pm 1$, $y = 0$, $y = \pi$, and $z = 0$.

475. Find the volume of the solid enclosed by the surface $z = x \sec^2 y$ and the planes z=0, x=0, x=2, y=0, and $y=\pi/4$.

476. Find the volume of the solid in the first octant bounded by the cylinder $z=16-x^2$ and the plane y=5.

477. Find the volume of the solid enclosed by the paraboloid $z = 2 + x^2 + (y - 2)^2$ and the planes $z = 1$, $x = 1$, $x = -1$, $y = 0$, and $y = 4$.

Find the volume of the given solid.

478. Under the plane $x + 2y - z = 0$ and above the region bounded by $y = x$ and $y = x^4$.

479. Under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.

480. Under the surface $z = xy$ and above the triangle with vertices (1,1), (4,1), and (1,2).

481. Enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0$, $y = 1$, $y = x$, $z = 0$

482. Bounded by the coordinate planes and the plane $3x + 2y + z = 6$

483. Bounded by the planes $z = x$, $y = x$, $x + y = 2$, and $z = 0$.

484. Enclosed by the cylinders $z = x^2$, $y = x^2$ and the planes $z = 0$, $y = 4$

485. Bounded by the cylinder $x^2 + y^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant

486. Bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y = z$, $x = 0$, $z = 0$ in the first octant

487. Bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$

488. The solid enclosed by the parabolic cylinders $y = 1 - x^2$, $y = x^2 - 1$ and the planes $x + y + z = 2$, $2x + 2y - z + 10 = 0$

489. The solid enclosed by the parabolic cylinder $y = x^2$ and the planes $z = 3y$, $z = 2 + y$

490. Sketch the solid whose volume is given by the iterated integral.

$$\int_0^1 \int_0^{1-x} (1 - x - y)\,dy\,dx$$

491. Sketch the region of integration and change the order of integration.

$$\int_0^4 \int_0^{\sqrt{x}} f(x, y)\,dy\,dx$$