

Pg.11

9.

The perimeter is

$$P = 3x$$

The area of a triangle is

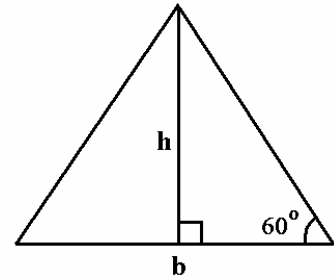
$$A = \frac{1}{2}bh$$

where b is the base, h is the height

$$h = \frac{1}{2}b \tan 60^\circ = \frac{1}{2}b \cdot \sqrt{3} = \frac{\sqrt{3}}{2}b$$

In our case $b = x$, then the area is

$$A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$



10. By Pythagorean theorem

$$a^2 + a^2 = d^2$$

$$2a^2 = d^2$$

$$a^2 = \frac{d^2}{2}$$

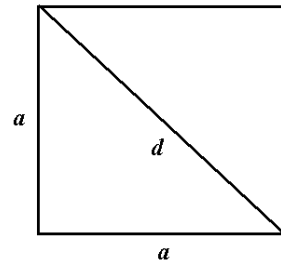
$$a = \sqrt{\frac{d^2}{2}}$$

$$a = \frac{d}{\sqrt{2}}$$

$$a = \frac{\sqrt{2}d}{2}$$

The area of the square is

$$A = a^2 = \left(\frac{\sqrt{2}d}{2}\right)^2 = \frac{2d^2}{4} = \frac{1}{2}d^2$$



11. From the triangle ABC by Pythagorean theorem
 $AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$

From the triangle ACD by Pythagorean theorem
 $AD^2 = AC^2 + CD^2 = 2a^2 + a^2 = 3a^2$

Thus,

$$d^2 = 3a^2$$

$$a^2 = \frac{d^2}{3}$$

$$a = \sqrt{\frac{d^2}{3}}$$

$$a = \frac{d}{\sqrt{3}}$$

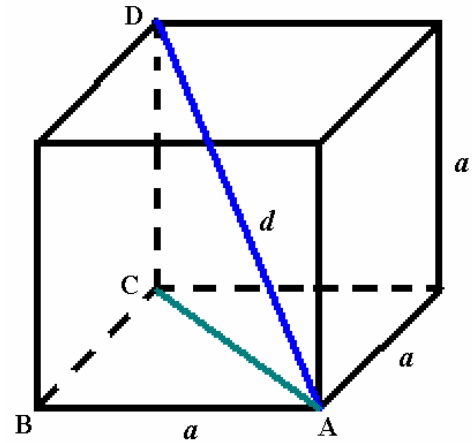
$$a = \frac{\sqrt{3}d}{3}$$

The surface area is

$$A = 6a^2 = 6\left(\frac{\sqrt{3}d}{3}\right)^2 = 6 \cdot \frac{d^2}{3} = 2d^2$$

The volume is

$$V = a^3 = \left(\frac{\sqrt{3}d}{3}\right)^3 = \frac{3\sqrt{3}d^3}{27} = \frac{\sqrt{3}d^3}{9}$$



23.

(a) $|y| = x$

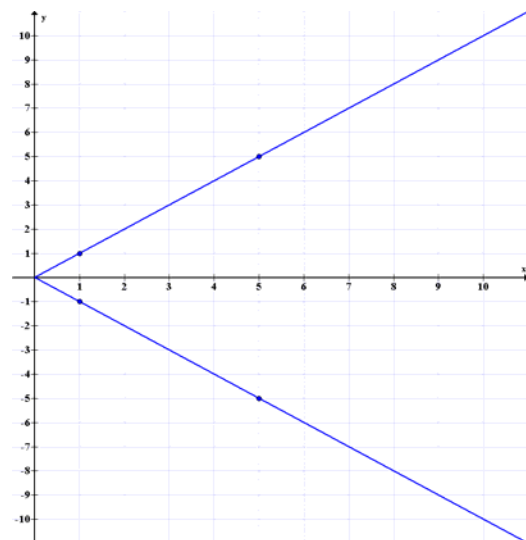
This equation can be rewritten as

$$x = \begin{cases} -y, & y < 0 \\ y, & y \geq 0 \end{cases}$$

The first line passes through the points
 $(1, -1)$ and $(5, -5)$.

The second line passes through the points
 $(1, 1)$ and $(5, 5)$.

The graph is not graph of a function
because we can draw a vertical line that
intersects it more than once.



(b) $y^2 = x^2$

This equation can be rewritten as

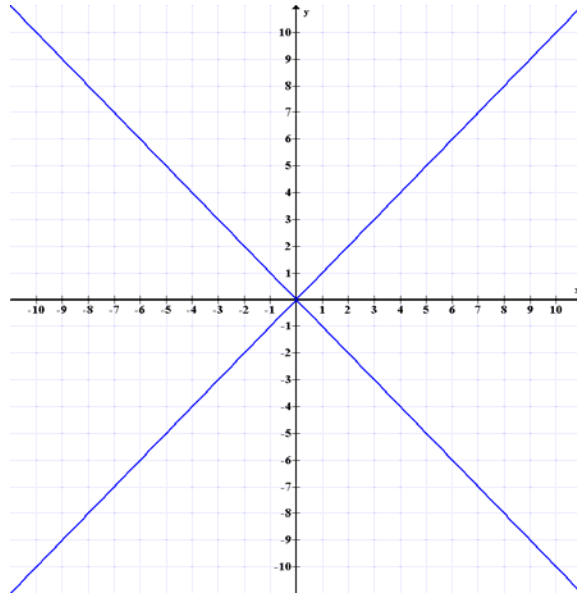
$$y^2 - x^2 = 0$$

$$(y - x)(y + x) = 0$$

$$y - x = 0 \quad \text{or} \quad y + x = 0$$

$$y = x \quad \text{or} \quad y = -x$$

The graph is not graph of a function because we can draw a vertical line that intersects it more than once.



24. (a) $|x| + |y| = 1$

In the Quadrant I $x > 0$ and $y > 0$, so we have

$$x + y = 1$$

$$y = -x + 1$$

This is a line with the slope $m = -1$ and the y-intercept $(0, 1)$.

In the Quadrant II $x < 0$ and $y > 0$, so we have

$$-x + y = 1$$

$$y = x + 1$$

This is a line with the slope $m = 1$ and the y-intercept $(0, 1)$.

In the Quadrant III $x < 0$ and $y < 0$, so we have

$$-x - y = 1$$

$$y = -x - 1$$

This is a line with the slope $m = -1$ and the y-intercept $(0, -1)$.

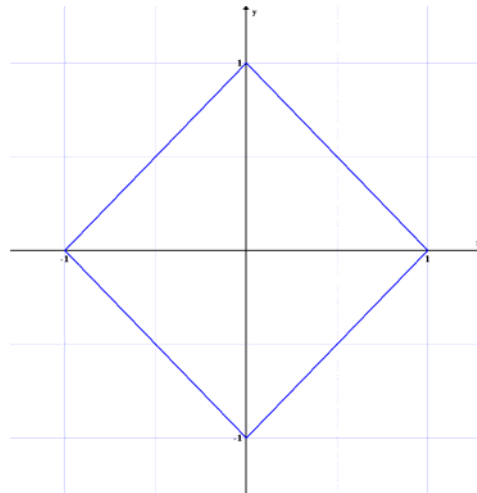
In the Quadrant IV $x > 0$ and $y < 0$, so we have

$$x - y = 1$$

$$y = x - 1$$

This is a line with the slope $m = 1$ and the y-intercept $(0, -1)$.

The graph is not graph of a function because we can draw a vertical line that intersects it more than once.



(b) $|x + y| = 1$

We have two equations:

$$x + y = 1 \text{ and } x + y = -1$$

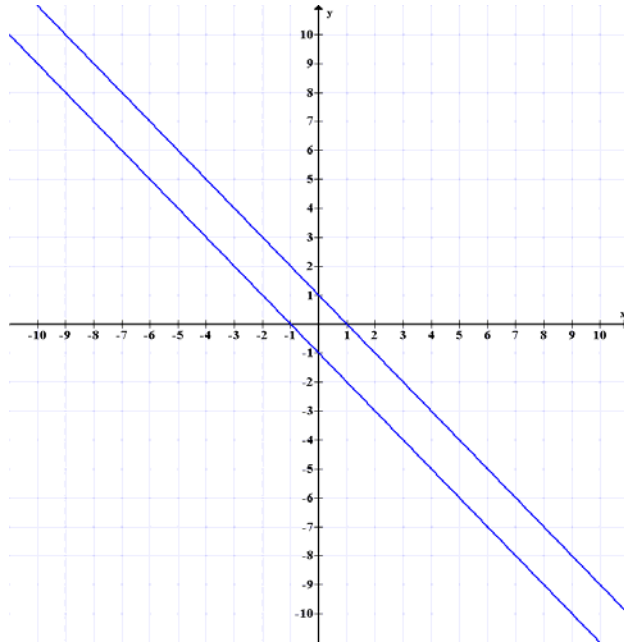
or

$$y = -x + 1 \text{ and } y = -x - 1$$

The first graph is a line with the slope $m = -1$ and the y-intercept $(0, 1)$.

The second graph is a line with the slope $m = -1$ and the y-intercept $(0, -1)$.

The graph is not graph of a function because we can draw a vertical line that intersects it more than once.



27.

$$F(x) = \begin{cases} 4 - x^2, & x \leq 1 \\ x^2 + 2x, & x > 1 \end{cases}$$

When $x \leq 1$ then $F(x) = 4 - x^2$, so the part of the graph of F that lies to the left of the vertical line $x = 1$ must coincide with the parabola $y = 4 - x^2$.

The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{0}{2(-1)} = 0$$

Find y for $x = 0$:

$$y = 4 - 0^2 = 4.$$

The vertex is $(0, 4)$.

Find few points:

x	1	-1	-2	-3	-4
$y = 4 - x^2$	3	3	0	-5	-12

When $x > 1$ then $F(x) = x^2 + 2x$, so the part of the graph of F that lies to the right of the vertical line $x = 1$ must coincide with the parabola $y = x^2 + 2x$.

The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{-2}{2 \cdot 1} = 1$$

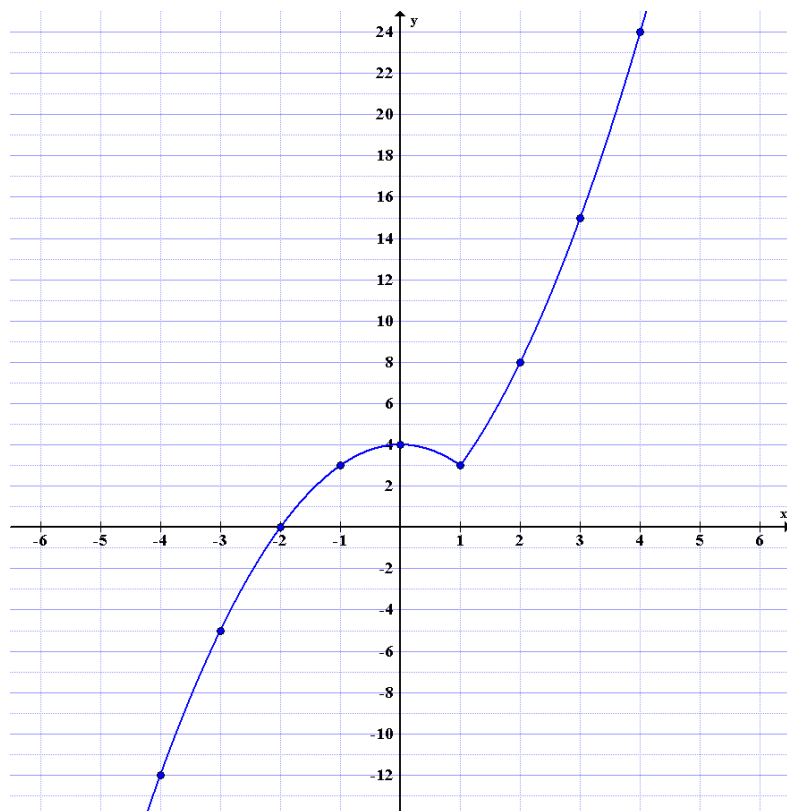
Find y for $x = 1$:

$$y = 1^2 + 2(1) = 3$$

The vertex is (1, 3).

Find few points:

x	2	3	4
$y = x^2 + 2x$	8	15	24



28.

$$G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & x > 0 \end{cases}$$

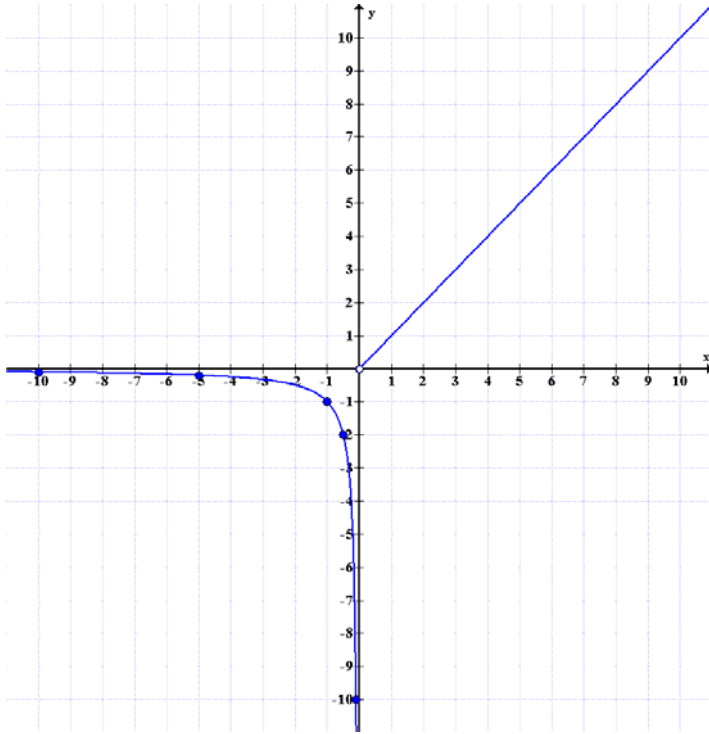
When $x < 0$ then $G(x) = 1/x$, so the part of the graph of F that lies to the left of the vertical line $x=0$ must coincide with the graph of $y = 1/x$.

Find few points:

x	-10	-5	-1	-0.5	-0.1
$y = 1/x$	-0.1	-0.2	-1	-2	-10

When $x > 0$ then $G(x) = x$, so the part of the graph of F that lies to the right of the vertical line $x=0$ must coincide with the graph of $y = x$.

This line have slope $m = 1$ and y -intercept $(0, 0)$.



29. (a) On the interval $[0, 1]$ the line passes through the point $(0, 0)$ and $(1, 1)$, so the equation of the graph on this interval is $y = x$.

On the interval $[1, 2]$ the line passes through the point $(1, 1)$ and $(2, 0)$.

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{2 - 1} = -1$$

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -(x - 1)$$

$$y - 1 = -x + 1$$

$$y = -x + 2$$

The equation of the function is

$$f(x) = \begin{cases} x, & 0 \leq x < 1 \\ -x + 2, & 1 \leq x \leq 2 \end{cases}$$

(b) The equation of the function is

$$f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$$

30. (a) On the interval $(0, 2]$ the line passes through the point $(0, 2)$ and $(2, 0)$.

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{2 - 0} = -1$$

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -(x - 0)$$

$$y = -x + 2$$

On the interval $(2, 5]$ the line passes through the point $(2, 1)$ and $(5, 0)$.

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{5 - 2} = -\frac{1}{3}$$

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{1}{3}(x - 5)$$

$$y = -\frac{1}{3}x + \frac{5}{3}$$

The equation of the function is

$$f(x) = \begin{cases} -x + 2, & 0 < x \leq 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \leq 5 \end{cases}$$

(b) On the interval $(-1, 0]$ the line passes through the point $(-1, 0)$ and $(0, -3)$.

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{0 - (-1)} = -3$$

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -3(x - (-1))$$

$$y = -3x - 3$$

On the interval $(0, 2]$ the line passes through the point $(0, 3)$ and $(2, -1)$.

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{2 - 0} = -2$$

The equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x - 0)$$

$$y = -2x + 3$$

The equation of the function is

$$f(x) = \begin{cases} -3x - 3, & -1 < x \leq 0 \\ -2x + 3, & 0 < x \leq 2 \end{cases}$$

Pg.19

11. $f(x) = x - 3$, $g(x) = \sqrt{x}$, $h(x) = x^3$, $j(x) = 2x$

(a)

$$y = \sqrt{x} - 3$$

$$y = (f \circ g)(x)$$

(b)

$$y = 2\sqrt{x}$$

$$y = (j \circ g)(x)$$

(c)

$$y = x^{\frac{1}{4}}$$

$$y = (g \circ g)(x)$$

(d)

$$y = 4x$$

$$y = (j \circ j)(x)$$

(e)

$$y = \sqrt{(x-3)^3}$$

$$(h \circ f)(x) = (x-3)^3$$

$$y = (g \circ h \circ f)(x)$$

(f)

$$y = (2x-6)^3$$

$$(j \circ f)(x) = (2x-6)$$

$$y = (h \circ j \circ f)(x)$$

12. (a)

$$y = 2x - 3$$

$$y = (f \circ j)(x)$$

(b)

$$y = x^{\frac{3}{2}}$$

$$y = (h \circ g)(x)$$

(c)

$$y = x^9$$

$$y = (h \circ h)(x)$$

(d)

$$y = x - 6$$

$$\frac{1}{4}h(x) = \frac{1}{2}x$$

$$2f(x) = 2x - 6$$

$$y = (2f \circ \frac{1}{4}h)(x)$$

(e)

$$y = 2\sqrt{x-3}$$

$$(g \circ f)(x) = \sqrt{x-3}$$

$$y = (h \circ g \circ f)(x)$$

(f)

$$y = \sqrt{x^3 - 3}$$

$$(f \circ h)(x) = x^3 - 3$$

$$y = (g \circ f \circ h)(x)$$

27. $y = x^3$, left 1, down 1

If $c > 0$ then

$y = f(x + c)$ shifts the graph of $y = f(x)$ a distance c units to the left

$y = f(x) - c$ shifts the graph of $y = f(x)$ a distance c units downward

Therefore,

$$y = (x + 1)^3$$

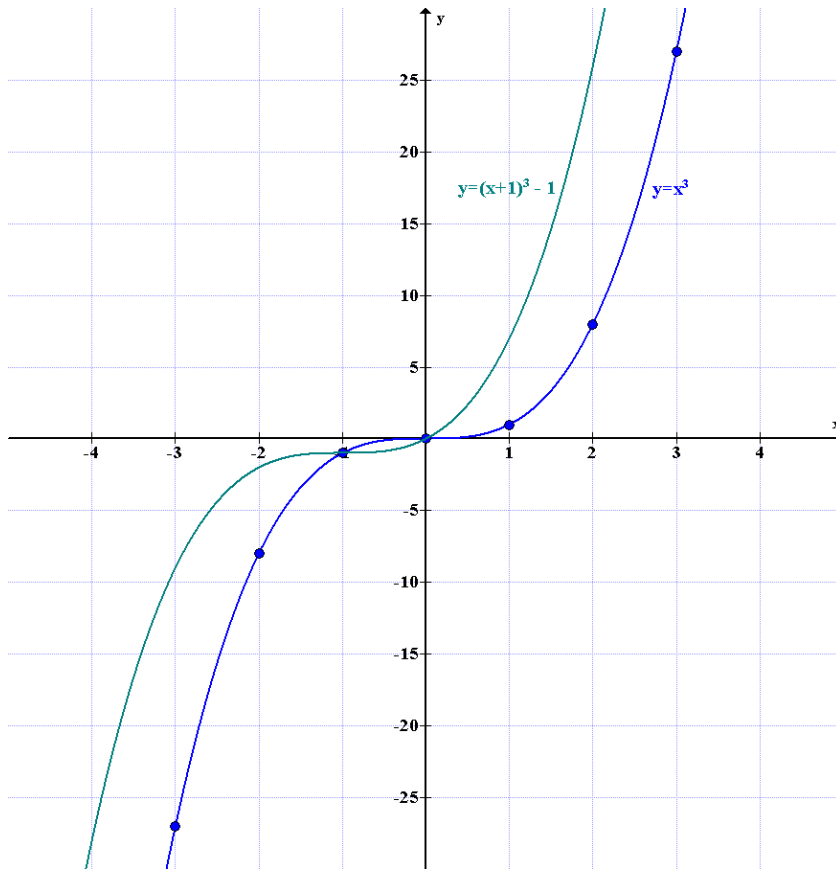
shifts the graph of $y = x^3$ one unit to the left

$$y = (x + 1)^3 - 1$$

shifts the graph of $y = (x + 1)^3$ one unit downward

Find few points:

x	-3	-2	-1	0	1	2	3
$y = x^3$	-27	-8	-1	0	1	8	27



28. $y = x^{\frac{2}{3}}$, right 1, down 1

If $c > 0$ then

$y = f(x - c)$ shifts the graph of $y = f(x)$ a distance c units to the left

$y = f(x) - c$ shifts the graph of $y = f(x)$ a distance c units downward

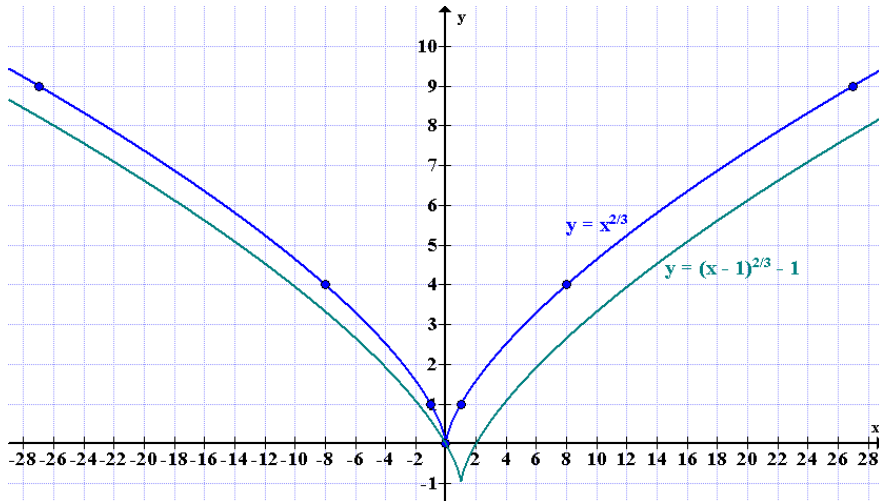
Therefore,

$y = (x - 1)^{\frac{2}{3}}$ shifts the graph of $y = x^{\frac{2}{3}}$ one unit to the right

$y = (x - 1)^{\frac{2}{3}} - 1$ shifts the graph of $y = (x - 1)^{\frac{2}{3}}$ one unit downward

Find few points:

x	-27	-8	-1	0	1	8	27
$y = x^{\frac{2}{3}}$	9	4	1	0	1	4	9

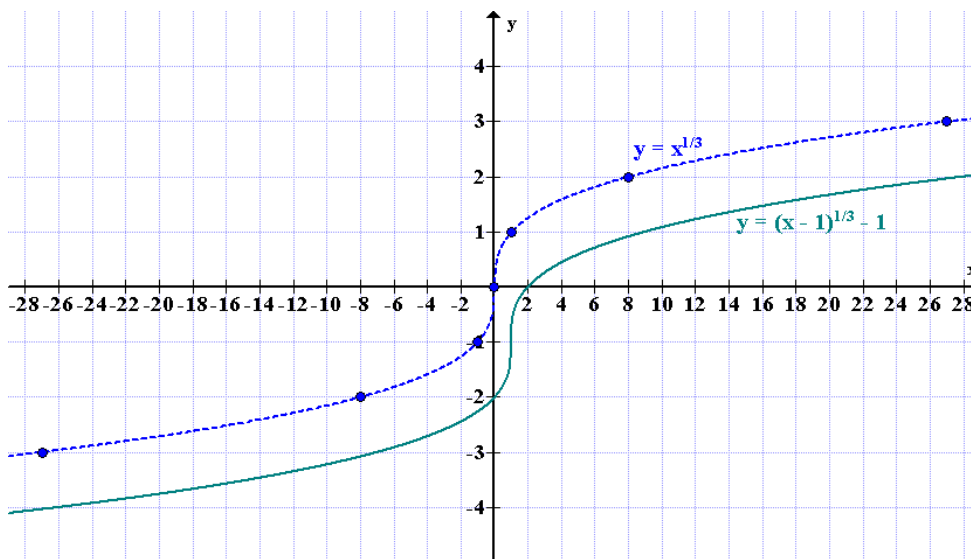


45. $y = \sqrt[3]{x-1} - 1$

The graph of $y = \sqrt[3]{x-1} - 1$ is the graph of $y = \sqrt[3]{x}$ shifted by 1 unit to the right and then moved by 1 unit downward.

Find few points:

x	-27	-8	-1	0	1	8	27
$y = \sqrt[3]{x}$	-3	-2	-1	0	1	2	3

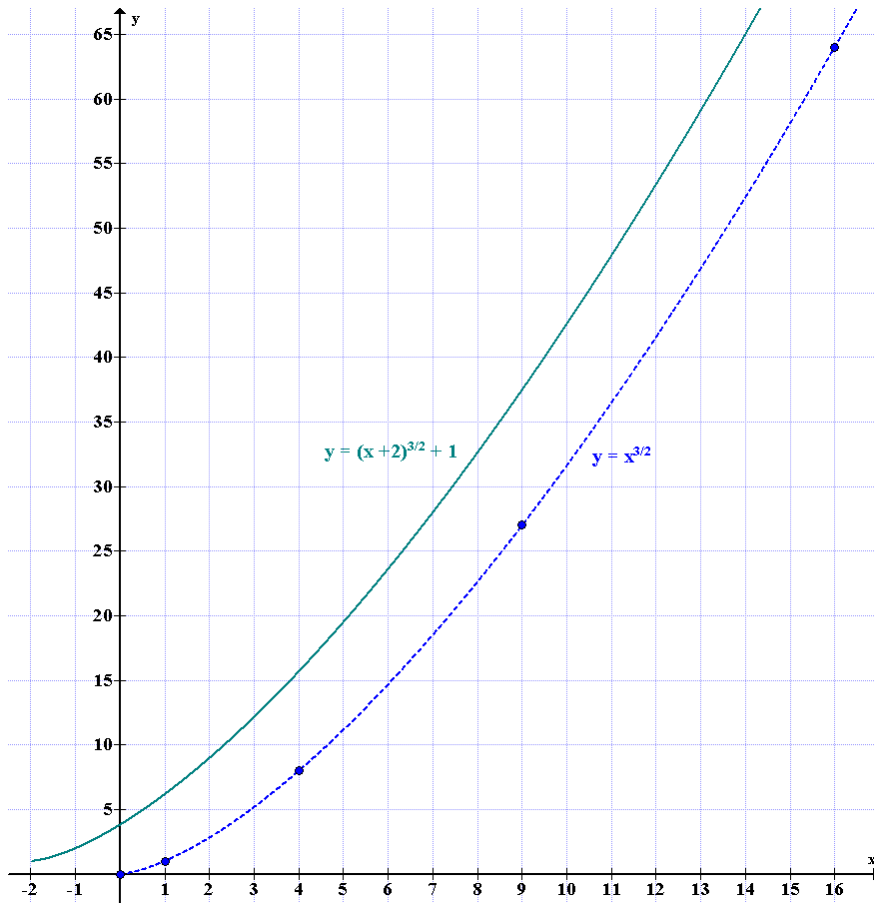


46. $y = (x + 2)^{3/2} + 1$

The graph of $y = (x + 2)^{3/2} + 1$ is the graph of $y = x^{3/2}$ shifted by 2 unit to the left and then moved by 1 unit upward.

Find few points:

x	0	1	4	9	16
$y = x^{3/2}$	0	1	8	27	64



59. $y = 1 + \frac{1}{x^2}$, compressed vertically by a factor of 2

If $c > 1$ then

$y = \frac{1}{c} f(x)$ compress the graph of $y = f(x)$ vertically by a factor of c .

Then

$$y = \frac{1}{2} \left(1 + \frac{1}{x^2} \right)$$

$$y = \frac{1}{2} + \frac{1}{2x^2}$$

60. $y = 1 + \frac{1}{x^2}$, stretched horizontally by a factor of 3

If $c > 1$ then

$y = f\left(\frac{x}{c}\right)$ stretch the graph of $y = f(x)$ horizontally by a factor of c .

Then

$$y = 1 + \frac{1}{\left(\frac{x}{3}\right)^2}$$

$$y = 1 + \frac{1}{\left(\frac{x^2}{9}\right)}$$

$$y = 1 + \frac{9}{x^2}$$

Pg.28

9. $\cos x = \frac{1}{3}$, $x \in \left[-\frac{\pi}{2}, 0\right]$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin x = \pm \sqrt{1 - \cos^2 x}$$

Since x is in Quadrant IV then

$$\sin x = -\sqrt{1 - \cos^2 x}$$

$$\sin x = -\sqrt{1 - \left(\frac{1}{3}\right)^2} = -\sqrt{1 - \frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{\sqrt{8}}{3} = -\frac{2\sqrt{2}}{3}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = -2\sqrt{2}$$

$$10. \cos x = -\frac{5}{13}, x \in \left[\frac{\pi}{2}, \pi \right]$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin x = \pm \sqrt{1 - \cos^2 x}$$

Since x is in Quadrant II then

$$\sin x = \sqrt{1 - \cos^2 x}$$

$$\sin x = \sqrt{1 - \left(-\frac{5}{13}\right)^2} = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{12}{13}}{-\frac{5}{13}} = -\frac{12}{5}$$

$$11. \tan x = \frac{1}{2}, x \in \left[\pi, \frac{3\pi}{2} \right]$$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \frac{1}{\cos^2 x}$$

$$\cos^2 x = \frac{1}{\tan^2 x + 1}$$

$$\cos x = \pm \sqrt{\frac{1}{\tan^2 x + 1}}$$

$$\cos x = \pm \frac{1}{\sqrt{\tan^2 x + 1}}$$

Since x is in Quadrant III then

$$\cos x = -\frac{1}{\sqrt{\tan^2 x + 1}}$$

$$\cos x = -\frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 + 1}} = -\frac{1}{\sqrt{\frac{1}{4} + 1}} = -\frac{1}{\sqrt{\frac{5}{4}}} = -\frac{\sqrt{4}}{\sqrt{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\sin x = \tan x \cos x = \frac{1}{2} \left(-\frac{2\sqrt{5}}{5} \right) = -\frac{\sqrt{5}}{5}$$

19. $y = \cos\left(x - \frac{\pi}{2}\right)$

The graph of $y = \cos\left(x - \frac{\pi}{2}\right)$ is the graph of $y = \cos x$ shifted by $\frac{\pi}{2}$ units to the right.

Find the y-intercept:

$$y = \cos\left(0 - \frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$$

The y-intercept is (0, 0).

Find the x-intercepts:

$$\cos\left(x - \frac{\pi}{2}\right) = 0$$

$$x - \frac{\pi}{2} = \frac{\pi}{2} + \pi k \quad k \in \mathbb{Z}$$

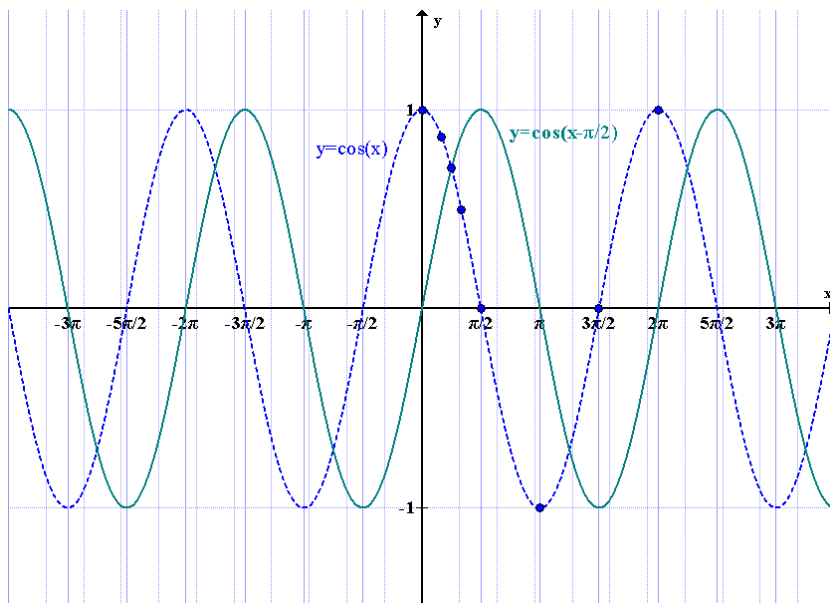
$$x = \pi + \pi k$$

The x-intercepts are $(\pi + \pi k, 0)$.

Find few points:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1

$y = \cos x$ is an even function so its graph is symmetric about the y-axis.



The period is 2π .

20. $y = \sin\left(x + \frac{\pi}{6}\right)$

The graph of $y = \sin\left(x + \frac{\pi}{6}\right)$ is the graph of $y = \sin x$ shifted by $\frac{\pi}{6}$ units to the left.

Find the y-intercept:

$$y = \sin\left(0 + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

The y-intercept is $\left(0, \frac{1}{2}\right)$.

Find the x-intercepts:

$$\sin\left(x + \frac{\pi}{6}\right) = 0$$

$$x + \frac{\pi}{6} = \pi k \quad k \in \mathbb{Z}$$

$$x = -\frac{\pi}{6} + \pi k$$

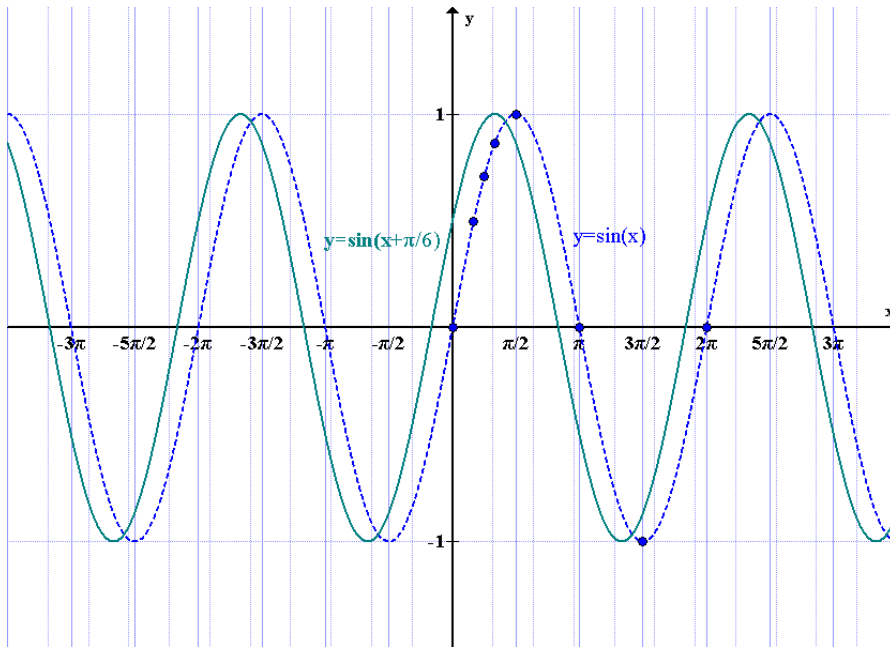
The x-intercepts are $\left(-\frac{\pi}{6} + \pi k, 0\right)$.

Find few points:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
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$y = \sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
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$y = \sin x$ is an odd function so its graph is symmetric about the origin.



The period is 2π .

$$31. \cos\left(x - \frac{\pi}{2}\right) = \sin x$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Transform the left side:

$$\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2}$$

$$= \cos x \cdot 0 + \sin x \cdot 1$$

$$= \sin x$$

$$32. \cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

Transform the left side:

$$\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos \frac{\pi}{2} - \sin x \sin \frac{\pi}{2}$$

$$= \cos x \cdot 0 - \sin x \cdot 1$$

$$= -\sin x$$

47. $\cos^2 \frac{\pi}{8}$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\cos^2 \frac{\pi}{8} = \frac{1 + \cos \frac{\pi}{4}}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4}$$

48. $\cos^2 \frac{\pi}{8}$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\cos^2 \frac{5\pi}{12} = \frac{1 + \cos \frac{5\pi}{6}}{2} = \frac{1 + \cos\left(\pi - \frac{\pi}{6}\right)}{2} = \frac{1 + \cos \pi \cos \frac{\pi}{6} + \sin \pi \sin \frac{\pi}{6}}{2}$$

$$= \frac{1 + (-1) \cdot \frac{\sqrt{3}}{2} + 0 \cdot \frac{1}{2}}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2}$$

$$= \frac{2 - \sqrt{3}}{4}$$

53. $\sin 2\theta - \cos \theta = 0$

$$\sin 2\theta - \cos \theta = 0$$

$$2 \sin \theta \cos \theta - \cos \theta = 0$$

$$\cos \theta (2 \sin \theta - 1) = 0$$

$$\cos \theta = 0 \quad \text{or} \quad 2 \sin \theta - 1 = 0$$

$$\cos \theta = 0 \quad \text{or} \quad \sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{2} + \pi k \quad k \in \mathbb{Z} \quad \text{or} \quad \theta = (-1)^m \sin^{-1} \frac{1}{2} + m\pi \quad m \in \mathbb{Z}$$

$$\theta = \frac{\pi}{2} + \pi k \quad \text{or} \quad \theta = (-1)^m \frac{\pi}{6} + \pi m$$

54. $\cos 2\theta + \cos \theta = 0$

$$\cos^2 \theta - \sin^2 \theta + \cos \theta = 0$$

$$\cos^2 \theta - (1 - \cos^2 \theta) + \cos \theta = 0$$

$$\cos^2 \theta - 1 + \cos^2 \theta + \cos \theta = 0$$

$$2 \cos^2 \theta - 1 + \cos \theta = 0$$

$$2 \cos^2 \theta + \cos \theta - 1 = 0$$

$$2 \cos^2 \theta + 2 \cos \theta - \cos \theta - 1 = 0$$

$$2 \cos \theta (\cos \theta + 1) - (\cos \theta + 1) = 0$$

$$(\cos \theta + 1)(2 \cos \theta - 1) = 0$$

$$\cos \theta + 1 = 0 \quad \text{or} \quad 2 \cos \theta - 1 = 0$$

$$\cos \theta = -1 \quad \text{or} \quad 2 \cos \theta = 1$$

$$\cos \theta = -1 \quad \text{or} \quad \cos \theta = \frac{1}{2}$$

$$\theta = \pi + 2\pi k \quad k \in \mathbb{Z} \quad \text{or} \quad \theta = \pm \cos^{-1} \frac{1}{2} + 2\pi m \quad m \in \mathbb{Z}$$

$$\theta = \pi + 2\pi k \quad \text{or} \quad \theta = \pm \frac{\pi}{3} + 2\pi m$$

Pg.63

3. $h(t) = \cot t$

The average rate of change of a function $y = f(x)$ over the interval $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$.

(a) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$$\frac{\cot\left(\frac{3\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right)}{\frac{3\pi}{4} - \frac{\pi}{4}} = \frac{-1 - 1}{\frac{\pi}{2}} = \frac{-2}{\frac{\pi}{2}} = -\frac{4}{\pi}$$

(b) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

$$\frac{\cot\left(\frac{\pi}{2}\right) - \cot\left(\frac{\pi}{6}\right)}{\frac{\pi}{2} - \frac{\pi}{6}} = \frac{0 - \sqrt{3}}{\frac{2\pi}{6}} = -\frac{\sqrt{3}}{\frac{\pi}{3}} = -\frac{3\sqrt{3}}{\pi}$$

4. $g(t) = 2 + \cos t$

The average rate of change of a function $y = f(x)$ over the interval $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$.

(a) $[0, \pi]$

$$\frac{2 + \cos \pi - (2 + \cos 0)}{\pi - 0} = \frac{2 + (-1) - (2 + 1)}{\pi} = \frac{1 - 3}{\pi} = -\frac{2}{\pi}$$

(a) $[-\pi, \pi]$

$$\frac{2 + \cos \pi - (2 + \cos(-\pi))}{\pi - (-\pi)} = \frac{2 + (-1) - (2 + (-1))}{2\pi} = \frac{1 - 1}{2\pi} = 0$$

8. Example 3 is needed

9. Example 3 is needed

13. Example 3 is needed

14. Example 3 is needed

$$15. \lim_{x \rightarrow 2} \frac{x+3}{x+6} = \frac{\lim_{x \rightarrow 2} (x+3)}{\lim_{x \rightarrow 2} (x+6)} = \frac{2+3}{2+6} = \frac{5}{8}$$

$$16. \lim_{s \rightarrow 2/3} 3s(2s-1) = \lim_{s \rightarrow 2/3} 3 \cdot \lim_{s \rightarrow 2/3} s \cdot \lim_{s \rightarrow 2/3} (2s-1)$$

$$= 3 \cdot \frac{2}{3} \cdot \left(2 \cdot \frac{2}{3} - 1 \right) = 2 \cdot \left(\frac{4}{3} - 1 \right)$$

$$= 2 \cdot \frac{1}{3} = \frac{2}{3}$$

$$21. \lim_{h \rightarrow 0} \frac{3}{\sqrt{3h+1}+1} = \frac{\lim_{h \rightarrow 0} 3}{\lim_{h \rightarrow 0} (\sqrt{3h+1}+1)} = \frac{3}{\lim_{h \rightarrow 0} \sqrt{3h+1} + \lim_{h \rightarrow 0} 1}$$

$$= \frac{3}{\sqrt{\lim_{h \rightarrow 0} (3h)} + \lim_{h \rightarrow 0} 1 + 1} = \frac{3}{\sqrt{0+1}+1} = \frac{3}{2}$$

$$22. \lim_{h \rightarrow 0} \frac{\sqrt{5h+4}-2}{h} = \lim_{h \rightarrow 0} \frac{(\sqrt{5h+4}-2)(\sqrt{5h+4}+2)}{h(\sqrt{5h+4}+2)}$$

$$= \lim_{h \rightarrow 0} \frac{5h+4-4}{h(\sqrt{5h+4}+2)} = \lim_{h \rightarrow 0} \frac{5h}{h(\sqrt{5h+4}+2)}$$

$$= \lim_{h \rightarrow 0} \frac{5}{\sqrt{5h+4}+2} = \frac{5}{\sqrt{5 \cdot 0 + 4} + 2}$$

$$= \frac{5}{\sqrt{4}+2} = \frac{5}{4}$$

$$27. \lim_{t \rightarrow 1} \frac{t^2+t-2}{t^2-1} = \lim_{t \rightarrow 1} \frac{t^2+2t-t-2}{t^2-1} = \lim_{t \rightarrow 1} \frac{t(t+2)-(t+2)}{(t-1)(t+1)}$$

$$= \lim_{t \rightarrow 1} \frac{(t+2)(t-1)}{(t-1)(t+1)} = \lim_{t \rightarrow 1} \frac{t+2}{t+1}$$

$$= \frac{1+2}{1+1} = \frac{3}{2}$$

28.

$$\begin{aligned}
\lim_{t \rightarrow -1} \frac{t^2 + 3t + 2}{t^2 - t - 2} &= \lim_{t \rightarrow -1} \frac{t^2 + t + 2t + 2}{t^2 - 2t + t - 2} = \lim_{t \rightarrow -1} \frac{t(t+1) + 2(t+1)}{t(t-2) + (t-2)} \\
&= \lim_{t \rightarrow -1} \frac{(t+1)(t+2)}{(t-2)(t+1)} = \lim_{t \rightarrow -1} \frac{t+2}{t-2} = \frac{-1+2}{-1-2} \\
&= -\frac{1}{3}
\end{aligned}$$

33.

$$\begin{aligned}
\lim_{u \rightarrow 1} \frac{u^4 - 1}{u^3 - 1} &= \lim_{u \rightarrow 1} \frac{(u^2 - 1)(u^2 + 1)}{(u - 1)(u^2 + u + 1)} = \lim_{u \rightarrow 1} \frac{(u - 1)(u + 1)(u^2 + 1)}{(u - 1)(u^2 + u + 1)} \\
&= \lim_{u \rightarrow 1} \frac{(u + 1)(u^2 + 1)}{u^2 + u + 1} = \frac{(1 + 1)(1^2 + 1)}{1^2 + 1 + 1} \\
&= \frac{4}{3}
\end{aligned}$$

34.

$$\begin{aligned}
\lim_{v \rightarrow 2} \frac{v^3 - 8}{v^4 - 16} &= \lim_{v \rightarrow 2} \frac{(v - 2)(v^2 + 2v + 4)}{(v^2 - 4)(v^2 + 4)} = \lim_{v \rightarrow 2} \frac{(v - 2)(v^2 + 2v + 4)}{(v - 2)(v + 2)(v^2 + 4)} \\
&= \lim_{v \rightarrow 2} \frac{v^2 + 2v + 4}{(v + 2)(v^2 + 4)} = \frac{2^2 + 2 \cdot 2 + 4}{(2 + 2)(2^2 + 4)} \\
&= \frac{12}{32} = \frac{3}{8}
\end{aligned}$$

$$45. \lim_{x \rightarrow 0} \sec x = \lim_{x \rightarrow 0} \frac{1}{\cos x} = \frac{1}{\cos 0} = \frac{1}{1} = 1$$

41.

$$\begin{aligned}
\lim_{x \rightarrow -3} \frac{2 - \sqrt{x^2 - 5}}{x + 3} &= \lim_{x \rightarrow -3} \frac{(2 - \sqrt{x^2 - 5})(2 + \sqrt{x^2 - 5})}{(x + 3)(2 + \sqrt{x^2 - 5})} \\
&= \lim_{x \rightarrow -3} \frac{4 - (x^2 - 5)}{(x + 3)(2 + \sqrt{x^2 - 5})} = \lim_{x \rightarrow -3} \frac{4 - x^2 + 5}{(x + 3)(2 + \sqrt{x^2 - 5})}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow -3} \frac{9 - x^2}{(x+3)(2 + \sqrt{x^2 - 5})} = \lim_{x \rightarrow -3} \frac{(3-x)(3+x)}{(x+3)(2 + \sqrt{x^2 - 5})} \\
&= \lim_{x \rightarrow -3} \frac{3-x}{2 + \sqrt{x^2 - 5}} = \frac{3 - (-3)}{2 + \sqrt{(-3)^2 - 5}} \\
&= \frac{6}{2 + \sqrt{4}} = \frac{6}{4} = \frac{3}{2}
\end{aligned}$$

49.

$$\begin{aligned}
\lim_{x \rightarrow -\pi} \sqrt{x+4} \cos(x+\pi) &= \lim_{x \rightarrow -\pi} \sqrt{x+4} \cdot \lim_{x \rightarrow -\pi} \cos(x+\pi) \\
&= \sqrt{-\pi+4} \cos(-\pi+\pi) = \sqrt{4-\pi} \cos 0 \\
&= \sqrt{4-\pi} \cdot 1 \\
&= \sqrt{4-\pi}
\end{aligned}$$

56. $\lim_{x \rightarrow -2} p(x) = 4$, $\lim_{x \rightarrow -2} r(x) = 0$, $\lim_{x \rightarrow -2} s(x) = -3$

$$(a) \lim_{x \rightarrow -2} (p(x) + r(x) + s(x)) = \lim_{x \rightarrow -2} p(x) + \lim_{x \rightarrow -2} r(x) + \lim_{x \rightarrow -2} s(x) = 4 + 0 + (-3) = 1$$

$$(b) \lim_{x \rightarrow -2} (p(x) \cdot r(x) \cdot s(x)) = \lim_{x \rightarrow -2} p(x) \cdot \lim_{x \rightarrow -2} r(x) \cdot \lim_{x \rightarrow -2} s(x) = 4 \cdot 0 \cdot (-3) = 0$$

(c)

$$\begin{aligned}
\lim_{x \rightarrow -2} \frac{(-4p(x) + 5r(x))}{s(x)} &= \frac{\lim_{x \rightarrow -2} (-4p(x) + 5r(x))}{\lim_{x \rightarrow -2} s(x)} = \frac{\lim_{x \rightarrow -2} (-4p(x)) + \lim_{x \rightarrow -2} (5r(x))}{-3} \\
&= \frac{-4 \lim_{x \rightarrow -2} p(x) + 5 \lim_{x \rightarrow -2} r(x)}{-3} = \frac{-4 \cdot 4 + 5 \cdot 0}{-3} = \frac{16}{3}
\end{aligned}$$

57. $f(x) = x^2$, $x = 1$

$$\begin{aligned}
m(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} (2x + h) = 2x
\end{aligned}$$

At $x = 1$ we have

$$m(1) = 2(1) = 2$$

58. $f(x) = x^2, x = -2$

$$\begin{aligned} m(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} (2x + h) = 2x \\ \text{At } x = -2 \text{ we have} \\ m(-2) &= 2(-2) = -4 \end{aligned}$$

61. $f(x) = \sqrt{x}, x = 7$

$$\begin{aligned} m(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\ &= \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}} \\ \text{At } x = 7 \text{ we have} \\ m(7) &= \frac{1}{2\sqrt{7}} = \frac{\sqrt{7}}{14} \end{aligned}$$

62. $f(x) = \sqrt{3x+1}, x = 0$

$$\begin{aligned} m(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{3(x+h)+1} - \sqrt{3x+1})(\sqrt{3(x+h)+1} + \sqrt{3x+1})}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h)+1 - (3x+1)}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})} = \lim_{h \rightarrow 0} \frac{3x+3h+1-3x-1}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})} \\ &= \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})} = \lim_{h \rightarrow 0} \frac{3}{\sqrt{3(x+h)+1} + \sqrt{3x+1}} \\ &= \frac{3}{\sqrt{3x+1} + \sqrt{3x+1}} = \frac{3}{2\sqrt{3x+1}} \\ \text{At } x = 0 \text{ we have} \\ m(0) &= \frac{3}{2\sqrt{3 \cdot 0 + 1}} = \frac{3}{2} \end{aligned}$$

9. It follows from the graph that

$$\text{if } \frac{9}{16} < x < \frac{25}{16} \text{ then } \frac{3}{4} < \sqrt{x} < \frac{5}{4}.$$

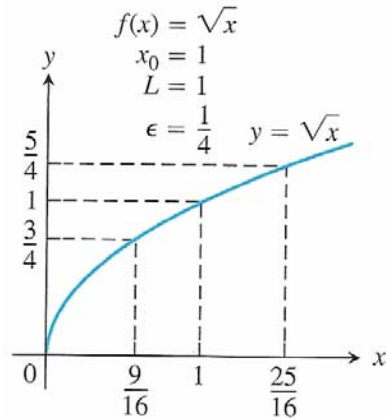
The interval $\left(\frac{9}{16}, \frac{25}{16}\right)$ is not symmetric about $x = 1$:

$$\frac{25}{16} - 1 = \frac{9}{16}$$

$$1 - \frac{9}{16} = \frac{7}{16}$$

We can choose δ to be the smaller of these numbers, that is

$$\delta = \frac{7}{16}.$$



10. It follows from the graph that

$$\text{if } 2.61 < x < 3.41 \text{ then } 3.8 < 2\sqrt{x+1} < 4.2$$

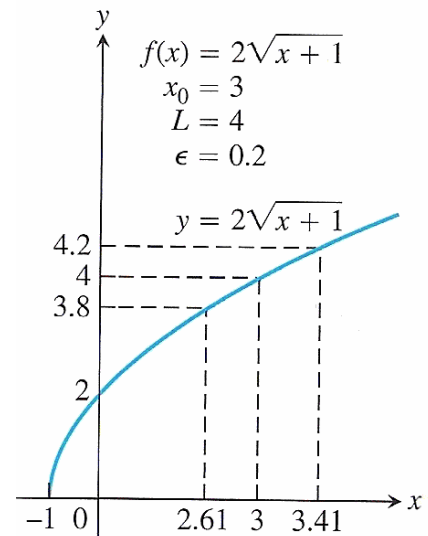
The interval $(2.61, 3.41)$ is not symmetric about $x = 3$:

$$3.41 - 3 = 0.41$$

$$3 - 2.61 = 0.39$$

We can choose δ to be the smaller of these numbers, that

$$\text{is } \delta = 0.39$$



20. $f(x) = \sqrt{x-7}$, $L = 4$, $x_0 = 23$, $\varepsilon = 1$

Step 1: Solve the inequality $|f(x) - L| < \varepsilon$ to find the interval about $x_0 = 23$ on which the inequality holds for all $x \neq x_0$:

$$\begin{aligned} |\sqrt{x-7} - 4| &< 1 \\ -1 &< \sqrt{x-7} - 4 < 1 \\ 3 &< \sqrt{x-7} < 5 \\ 9 &< x-7 < 25 \\ 16 &< x < 32 \end{aligned}$$

Step 2: Find a value of $\delta > 0$ that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval $16 < x < 32$:

$$\begin{aligned} 32 - 23 &= 9 \\ 23 - 16 &= 7 \end{aligned}$$

The distance from $x_0 = 23$ to the nearer endpoint of $(16, 32)$ is 7. If we take $\delta = 7$ or any smaller positive number, then

$$0 < |x - 23| < 7 \quad \Rightarrow \quad |\sqrt{x-7} - 4| < 1$$

21. $f(x) = \frac{1}{x}$, $L = \frac{1}{4}$, $x_0 = 4$, $\varepsilon = 0.05$

Step 1: Solve the inequality $|f(x) - L| < \varepsilon$ to find the interval about $x_0 = 4$ on which the inequality holds for all $x \neq x_0$:

$$\begin{aligned} \left| \frac{1}{x} - \frac{1}{4} \right| &< 0.05 \\ -0.05 &< \frac{1}{x} - \frac{1}{4} < 0.05 \\ 0.2 &< \frac{1}{x} < 0.3 \\ \frac{10}{3} &< x < 5 \end{aligned}$$

Step 2: Find a value of $\delta > 0$ that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the

interval $\frac{10}{3} < x < 5$:

$$5 - 4 = 1$$

$$4 - \frac{10}{3} = \frac{2}{3}$$

The distance from $x_0 = 4$ to the nearer endpoint of $\left(\frac{10}{3}, 5\right)$ is $\frac{2}{3}$. If we take $\delta = \frac{2}{3}$ or any smaller positive number, then

$$0 < |x - 4| < \frac{2}{3} \quad \Rightarrow \quad \left| \frac{1}{x} - \frac{1}{4} \right| < 0.05$$

25. $f(x) = x^2 - 5$, $L = 11$, $x_0 = 4$, $\varepsilon = 1$

Step 1: Solve the inequality $|f(x) - L| < \varepsilon$ to find the interval about $x_0 = 4$ on which the inequality holds for all $x \neq x_0$:

$$\begin{aligned} |x^2 - 5 - 11| &< 1 \\ -1 &< x^2 - 16 < 1 \\ 15 &< x^2 < 17 \\ \sqrt{15} &< x < \sqrt{17} \end{aligned}$$

Step 2: Find a value of $\delta > 0$ that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval $\sqrt{15} < x < \sqrt{17}$:

$$\sqrt{17} - 4 \approx 0.1231$$

$$4 - \sqrt{15} \approx 0.1270$$

The distance from $x_0 = 4$ to the nearer endpoint of $(\sqrt{15}, \sqrt{17})$ is $\sqrt{17} - 4$. If we take $\delta = \sqrt{17} - 4$ or any smaller positive number, then

$$0 < |x - 4| < \sqrt{17} - 4 \quad \Rightarrow \quad |x^2 - 5 - 11| = |x^2 - 16| < 1$$

33. $f(x) = \frac{x^2 - 4}{x - 2}$, $x_0 = 2$, $\varepsilon = 0.05$

$$L = \lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \rightarrow 2} (x + 2) = 2 + 2 = 4$$

Step 1: Solve the inequality $|f(x) - L| < \varepsilon$ to find the interval about $x_0 = 4$ on which the inequality holds for all $x \neq x_0$:

$$\begin{aligned} \left| \frac{x^2 - 4}{x - 2} - 4 \right| &< 0.05 \\ -0.05 &< \frac{x^2 - 4}{x - 2} - 4 < 0.05 \end{aligned}$$

$$3.95 < \frac{x^2 - 4}{x - 2} < 4.05$$

$$3.95 < \frac{(x-2)(x+2)}{x-2} < 4.05$$

$$3.95 < x+2 < 4.05$$

$$1.95 < x < 2.05$$

Step 2: Find a value of $\delta > 0$ that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval $1.95 < x < 2.05$:

$$2.05 - 2 = 0.05$$

$$2 - 1.95 = 0.05$$

The distance from $x_0 = 2$ to the nearer endpoint of $(1.95, 2.05)$ is 0.05. If we take $\delta = 0.05$ or any smaller positive number, then

$$0 < |x - 2| < 0.05 \quad \Rightarrow \quad \left| \frac{x^2 - 4}{x - 2} - 4 \right| < 0.05$$

34. $f(x) = \frac{x^2 + 6x + 5}{x + 5}$, $x_0 = -5$, $\varepsilon = 0.05$

$$L = \lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow -5} \frac{x^2 + 6x + 5}{x + 5} = \lim_{x \rightarrow -5} \frac{x^2 + x + 5x + 5}{x + 5} = \lim_{x \rightarrow -5} \frac{x(x+1) + 5(x+1)}{x + 5}$$

$$= \lim_{x \rightarrow -5} \frac{(x+1)(x+5)}{x + 5} = \lim_{x \rightarrow -5} (x+1) = -5 + 1 = -4$$

Step 1: Solve the inequality $|f(x) - L| < \varepsilon$ to find the interval about $x_0 = -5$ on which the inequality holds for all $x \neq x_0$:

$$\left| \frac{x^2 + 6x + 5}{x + 5} - (-4) \right| < 0.05$$

$$-0.05 < \frac{x^2 + 6x + 5}{x + 5} + 4 < 0.05$$

$$-4.05 < \frac{(x+5)(x+1)}{x+5} < -3.95$$

$$-4.05 < x+1 < -3.95$$

$$-5.05 < x < -4.95$$

Step 2: Find a value of $\delta > 0$ that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval $-5.05 < x < -4.95$:

$$-5 - (-5.05) = 0.05$$

$$-4.95 - (-5) = 0.05$$

The distance from $x_0 = -5$ to the nearer endpoint of $(-5.05, -4.95)$ is 0.05. If we take $\delta = 0.05$ or any smaller positive number, then

$$0 < |x+5| < 0.05 \quad \Rightarrow \quad \left| \frac{x^2 + 6x + 5}{x+5} + 4 \right| < 0.05$$

43. $\lim_{x \rightarrow 1} \frac{1}{x} = 1$

Given $\varepsilon > 0$, we want to find $\delta > 0$ such that

if $0 < |x-1| < \delta$ then $\left| \frac{1}{x} - 1 \right| < \varepsilon$

We have

$$\left| \frac{1}{x} - 1 \right| = \left| \frac{1-x}{x} \right| = \frac{|1-x|}{|x|} < \varepsilon$$

We find a positive constant C such that $\frac{1}{|x|} < C \Rightarrow \frac{|x-1|}{|x|} < C|x-1|$ and we can make

$$C|x-1| < \varepsilon \text{ by taking } |x-1| < \frac{\varepsilon}{C} = \delta$$

We restrict x to lie in the interval $|x-1| < \frac{1}{2}$, then

$$\frac{1}{2} < x < \frac{3}{2} \quad \Rightarrow \quad \frac{2}{3} < \frac{1}{x} < 2 \quad \Rightarrow \quad \frac{1}{|x|} < 2$$

So $C = 2$ is suitable and we should choose $\delta = \min \left\{ \frac{1}{2}, \frac{\varepsilon}{2} \right\}$.

Show that δ works.

Given $\varepsilon > 0$, we let $\delta = \min \left\{ \frac{1}{2}, \frac{\varepsilon}{2} \right\}$. If $0 < |x-1| < \delta$ then

$$|x-1| < \frac{1}{2} \quad \Rightarrow \quad \frac{1}{2} < x < \frac{3}{2} \quad \Rightarrow \quad \frac{1}{|x|} < 2$$

Also

$$\frac{|1-x|}{|x|} < \varepsilon \quad \Rightarrow \quad 2|1-x| < \varepsilon \quad \Rightarrow \quad |1-x| < \frac{\varepsilon}{2}$$

so

$$\left| \frac{1}{x} - 1 \right| = \frac{|1-x|}{|x|} < 2 \cdot \frac{\varepsilon}{2} = \varepsilon$$

Therefore,

$$\lim_{x \rightarrow 1} \frac{1}{x} = 1$$

$$44. \lim_{x \rightarrow \sqrt{3}} \frac{1}{x^2} = \frac{1}{3}$$

Given $\varepsilon > 0$, we want to find $\delta > 0$ such that

$$\text{if } 0 < |x - \sqrt{3}| < \delta \quad \text{then} \quad \left| \frac{1}{x^2} - \frac{1}{3} \right| < \varepsilon$$

We have

$$\left| \frac{1}{x^2} - \frac{1}{3} \right| = \left| \frac{3 - x^2}{3x^2} \right| = \left| \frac{(\sqrt{3} - x)(\sqrt{3} + x)}{3x^2} \right| = \frac{|\sqrt{3} - x| |\sqrt{3} + x|}{|3x^2|} < \varepsilon$$

Let's assume that $\delta \leq 1$. This is a valid assumption to make since, in general, once we find a δ that works, all smaller values of δ also work. Then $|x - \sqrt{3}| < \delta \leq 1$ implies that

$$-1 < x - \sqrt{3} < 1$$

$$-1 + \sqrt{3} < x < 1 + \sqrt{3}$$

$$-1 + 2\sqrt{3} < x + \sqrt{3} < 1 + 2\sqrt{3}$$

Then

$$|\sqrt{3} - x| |\sqrt{3} + x| < |\sqrt{3} - x| (1 + 2\sqrt{3}) < \varepsilon$$

$$|\sqrt{3} - x| < \frac{\varepsilon}{(1 + 2\sqrt{3})}$$

Now choose $\delta = \min \left\{ 1, \frac{\varepsilon}{(1 + 2\sqrt{3})} \right\}$. This guarantees that both assumptions made about δ in

the course of this proof are taken into account simultaneously. Thus, if $0 < |x - \sqrt{3}| < \delta$, it

follows that $\left| \frac{1}{x^2} - \frac{1}{3} \right| < \varepsilon$. This completes the proof.

7.

(a)

$$f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

Find the x-intercept:

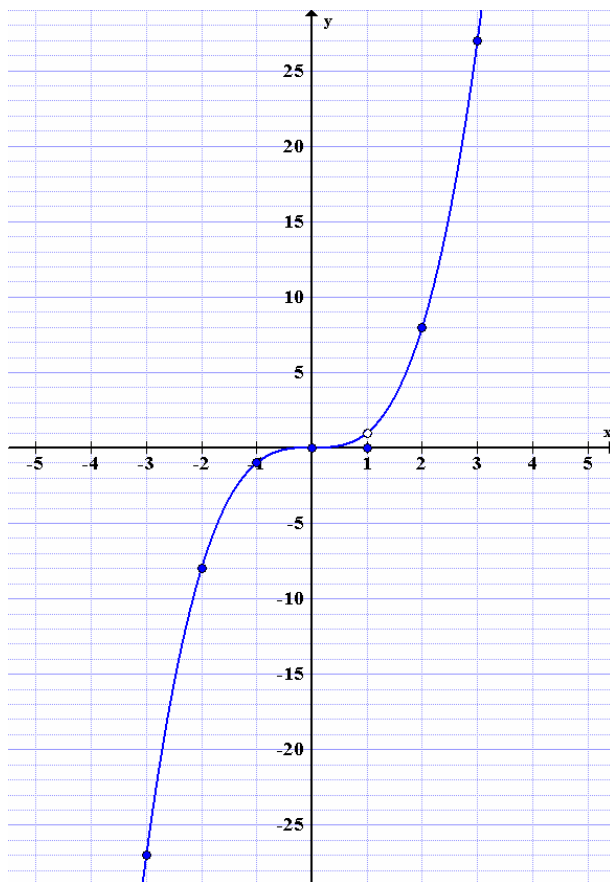
$$x^3 = 0$$

$$x = 0$$

The x-intercept is (0, 0).

Find few points:

x	-3	-2	-1	1	2	3
$y = x^3$	-27	-8	-1	1	8	27



(b)

$$\lim_{x \rightarrow 1^-} f(x) = 1$$

$$\lim_{x \rightarrow 1^+} f(x) = 1$$

(c) Since $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$ then $\lim_{x \rightarrow 1} f(x)$ exists:

$$\lim_{x \rightarrow 1} f(x) = 1$$

$$8. f(x) = \begin{cases} 1 - x^2, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

(a) Find the x-intercepts:

$$1 - x^2 = 0$$

$$(1 - x)(1 + x) = 0$$

$$1 - x = 0 \quad \text{or} \quad 1 + x = 0$$

$$x = 1 \quad \text{or} \quad x = -1$$

The x-intercept is $(-1, 0)$ (the point $(1, 0)$ is not on the graph of f).

Find the y-intercept:

$$f(0) = 1 - 0^2 = 1$$

The y-intercept is $(0, 1)$.

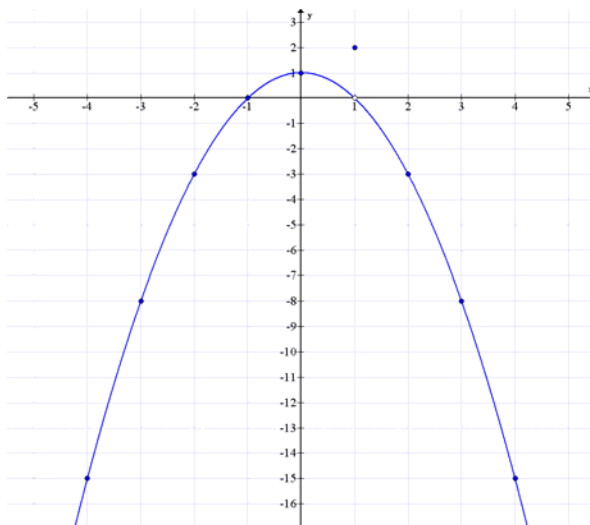
The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{0}{2 \cdot (-1)} = 0$$

The vertex is $(0, 1)$.

Find few points:

x	-4	-3	-2	2	3	4
$y = x^3$	-15	-8	-3	-3	-8	-15



(b)

$$\lim_{x \rightarrow 1^-} f(x) = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = 0$$

(c) Since $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$ then $\lim_{x \rightarrow 1} f(x)$ exists:

$$\lim_{x \rightarrow 1} f(x) = 0$$

$$11. \lim_{x \rightarrow -0.5^-} \sqrt{\frac{x+2}{x+1}} = \sqrt{\frac{-0.5+2}{-0.5+1}} = \sqrt{\frac{1.5}{0.5}} = \sqrt{3}$$

$$12. \lim_{x \rightarrow 1^+} \sqrt{\frac{x-1}{x+2}}$$

As $x \rightarrow 1^+$, we have $(x-1) \rightarrow 0^+$ and $(x+2) \rightarrow 3^+$, so

$$\lim_{x \rightarrow 1^+} \sqrt{\frac{x-1}{x+2}} = 0$$

13.

$$\begin{aligned} \lim_{x \rightarrow -2^+} \left(\frac{x}{x+1} \right) \left(\frac{2x+5}{x^2+x} \right) &= \lim_{x \rightarrow -2^+} \frac{x(2x+5)}{(x+1)(x^2+x)} = \lim_{x \rightarrow -2^+} \frac{x(2x+5)}{x(x+1)(x+1)} \\ &= \lim_{x \rightarrow -2^+} \frac{2x+5}{(x+1)^2} = \frac{2(-2)+5}{(-2+1)^2} = \frac{1}{(-1)^2} = 1 \end{aligned}$$

25.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\tan 2x}{x} &= \lim_{x \rightarrow 0} \frac{\sin 2x}{\cos 2x} = \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x} \cdot \frac{1}{\cos 2x} \right) = 2 \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \cdot \frac{1}{\cos 2x} \right) \\ &= 2 \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos 2x} = 2 \cdot 1 \cdot \frac{1}{\cos 0} = 2 \cdot \frac{1}{1} \\ &= 2 \end{aligned}$$

26.

$$\begin{aligned} \lim_{t \rightarrow 0} \frac{2t}{\tan t} &= \lim_{t \rightarrow 0} \frac{2t}{\frac{\sin t}{\cos t}} = \lim_{t \rightarrow 0} \frac{2t \cos t}{\sin t} = \lim_{t \rightarrow 0} \frac{2 \cos t}{\frac{\sin t}{t}} \\ &= \frac{2 \lim_{t \rightarrow 0} \cos t}{\lim_{t \rightarrow 0} \frac{\sin t}{t}} = \frac{2 \cos 0}{1} = 2 \end{aligned}$$

39.

$$\begin{aligned}
\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 8x} &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin 3x}{\cos 3x}}{8x \cdot \frac{\sin 8x}{8x}} \right) = \lim_{x \rightarrow 0} \left(\frac{1}{8x} \frac{\sin 3x}{\sin 8x} \cdot \frac{1}{\cos 3x} \right) = \lim_{x \rightarrow 0} \left(\frac{1}{8x} \frac{3x \cdot \frac{\sin 3x}{3x}}{8} \cdot \frac{1}{\cos 3x} \right) \\
&= \lim_{x \rightarrow 0} \left(\frac{3x}{8x} \frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8x}} \cdot \frac{1}{\cos 3x} \right) = \lim_{x \rightarrow 0} \left(\frac{3}{8} \frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8x}} \cdot \frac{1}{\cos 3x} \right) = \frac{3}{8} \lim_{x \rightarrow 0} \left(\frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8x}} \cdot \frac{1}{\cos 3x} \right) \\
&= \frac{3}{8} \left(\lim_{x \rightarrow 0} \frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8x}} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos 3x} \right) = \frac{3}{8} \left(\frac{\lim_{x \rightarrow 0} \frac{\sin 3x}{3x}}{\lim_{x \rightarrow 0} \frac{\sin 8x}{8x}} \cdot \frac{1}{\cos 0} \right) \\
&= \frac{3}{8} \left(\frac{1}{1} \cdot \frac{1}{1} \right) = \frac{3}{8}
\end{aligned}$$

40.

$$\begin{aligned}
\lim_{y \rightarrow 0} \frac{\sin 3y \cot 5y}{y \cot 4y} &= \lim_{y \rightarrow 0} \left(\frac{\sin 3y}{y} \cdot \frac{\cot 5y}{\cot 4y} \right) = \lim_{y \rightarrow 0} \left(3 \cdot \frac{\sin 3y}{3y} \cdot \frac{\frac{\cos 5y}{\sin 5y}}{\frac{\cos 4y}{\sin 4y}} \right) \\
&= 3 \lim_{y \rightarrow 0} \left(\frac{\sin 3y}{3y} \cdot \frac{\cos 5y}{\sin 5y} \cdot \frac{\sin 4y}{\cos 4y} \right) = 3 \lim_{y \rightarrow 0} \left(\frac{\sin 3y}{3y} \cdot \frac{\sin 4y}{\sin 5y} \cdot \frac{\cos 5y}{\cos 4y} \right) \\
&= 3 \left(\lim_{y \rightarrow 0} \frac{\sin 3y}{3y} \cdot \lim_{y \rightarrow 0} \frac{\sin 4y}{\sin 5y} \cdot \lim_{y \rightarrow 0} \frac{\cos 5y}{\cos 4y} \right) = 3 \left(1 \cdot \lim_{y \rightarrow 0} \frac{4y \cdot \frac{\sin 4y}{4y}}{5y \cdot \frac{\sin 5y}{5y}} \cdot \frac{\cos 0}{\cos 0} \right) \\
&= 3 \left(\lim_{y \rightarrow 0} \left(\frac{4y}{5y} \frac{\frac{\sin 4y}{4y}}{\frac{\sin 5y}{5y}} \right) \cdot \frac{1}{1} \right) = 3 \lim_{y \rightarrow 0} \left(\frac{4}{5} \frac{\frac{\sin 4y}{4y}}{\frac{\sin 5y}{5y}} \right) = \frac{12}{5} \lim_{y \rightarrow 0} \left(\frac{\frac{\sin 4y}{4y}}{\frac{\sin 5y}{5y}} \right) \\
&= \frac{12}{5} \cdot \frac{\lim_{y \rightarrow 0} \frac{\sin 4y}{4y}}{\lim_{y \rightarrow 0} \frac{\sin 5y}{5y}} = \frac{12}{5} \cdot \frac{1}{1} = \frac{12}{5}
\end{aligned}$$

41.

$$\begin{aligned} \lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta^2 \cot 3\theta} &= \lim_{\theta \rightarrow 0} \frac{\frac{\sin \theta}{\cos \theta}}{\theta^2 \frac{\cos 3\theta}{\sin 3\theta}} = \lim_{\theta \rightarrow 0} \frac{\sin \theta \sin 3\theta}{\theta^2 \cos \theta \cos 3\theta} = \lim_{\theta \rightarrow 0} \left(\frac{\sin \theta}{\theta} \cdot \frac{\sin 3\theta}{\theta} \cdot \frac{1}{\cos \theta \cos 3\theta} \right) \\ &= \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \cdot \lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{\theta} \cdot \lim_{\theta \rightarrow 0} \frac{1}{\cos \theta \cos 3\theta} = 1 \cdot 3 \lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{3\theta} \cdot \frac{1}{\cos 0 \cos 0} \\ &= 3 \cdot 1 \cdot \frac{1}{1 \cdot 1} = 3 \end{aligned}$$

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13. $y = \frac{1}{x-2} - 3x$

The first term is a rational function so it is continuous everywhere except $x = 2$ (it is not defined at $x = 2$), the second term is a polynomial function so it is continuous everywhere. Therefore, y is continuous on the intervals $(-\infty, 2) \cup (2, \infty)$.

14. $y = \frac{1}{(x+2)^2} + 4$

The first term is a rational function so it is continuous everywhere except $x = -2$ (it is not defined at $x = -2$), the second term is a constant function so it is continuous everywhere. Therefore, y is continuous on the intervals $(-\infty, -2) \cup (-2, \infty)$.

15. $y = \frac{x+1}{x^2 - 4x + 3}$

Find the domain:

$$x^2 - 4x + 3 = 0$$

$$x^2 - 3x - x + 3 = 0$$

$$x(x-3) - (x-3) = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \quad \text{or} \quad x = 1$$

The domain is the intervals $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$.

y is a rational function so it is continuous on its domain or on the intervals $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$.

35.

$$\begin{aligned}\lim_{t \rightarrow 0} \cos\left(\frac{\pi}{\sqrt{19-3\sec 2t}}\right) &= \cos\left(\frac{\pi}{\sqrt{19-3\sec 0}}\right) = \cos\left(\frac{\pi}{\sqrt{19-3 \cdot 1}}\right) \\ &= \cos\left(\frac{\pi}{\sqrt{16}}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\end{aligned}$$

$$\cos\left(\frac{\pi}{\sqrt{19-3\sec 0}}\right) = \cos\left(\frac{\pi}{\sqrt{19-3 \cdot 1}}\right) = \cos\left(\frac{\pi}{\sqrt{16}}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Since $\lim_{t \rightarrow 0} f(t) = f(0)$ then the function is continuous at $t = 0$.

$$36. \lim_{x \rightarrow \frac{\pi}{6}} \sqrt{\csc^2 x + 5\sqrt{3} \tan x} = \sqrt{2^2 + 5\sqrt{3} \cdot \frac{\sqrt{3}}{3}} = \sqrt{4+5} = 3$$

$$\sqrt{\csc^2 x + 5\sqrt{3} \tan x} = \sqrt{2^2 + 5\sqrt{3} \cdot \frac{\sqrt{3}}{3}} = \sqrt{4+5} = 3$$

Since $\lim_{x \rightarrow \frac{\pi}{6}} f(x) = f\left(\frac{\pi}{6}\right)$ then the function is continuous at $x = \frac{\pi}{6}$.

$$37. \lim_{x \rightarrow 0^+} \sin\left(\frac{\pi}{2} e^{\sqrt{x}}\right) = \sin\left(\frac{\pi}{2} e^{\sqrt{0}}\right) = \sin\left(\frac{\pi}{2} \cdot 1\right) = 1$$

$$\sin\left(\frac{\pi}{2} e^{\sqrt{0}}\right) = \sin\left(\frac{\pi}{2} \cdot 1\right) = 1$$

Since $\lim_{x \rightarrow 0^+} f(x) = f(0)$ then the function is continuous from the right at $x = 0$.

$$39. g(x) = \frac{x^2 - 9}{x - 3}$$

Find one-sided limits at $x = 3$:

$$\lim_{x \rightarrow 3^-} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3^-} \frac{(x-3)(x+3)}{x-3} = \lim_{x \rightarrow 3^-} (x+3) = 3+3 = 6$$

$$\lim_{x \rightarrow 3^+} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3^+} \frac{(x-3)(x+3)}{x-3} = \lim_{x \rightarrow 3^+} (x+3) = 3+3 = 6$$

Thus,

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$$

To be continuous at $x = 3$ we must define $g(3) = 6$.

$$40. h(t) = \frac{t^2 + 3t - 10}{t - 2}$$

Find one-sided limits at $t = 2$:

$$\begin{aligned} \lim_{t \rightarrow 2^-} \frac{t^2 + 3t - 10}{t - 2} &= \lim_{t \rightarrow 2^-} \frac{t^2 + 5t - 2t - 10}{t - 2} = \lim_{t \rightarrow 2^-} \frac{t(t+5) - 2(t+5)}{t - 2} \\ &= \lim_{t \rightarrow 2^-} \frac{(t+5)(t-2)}{t-2} = \lim_{t \rightarrow 2^-} (t+5) = 2+5 = 7 \end{aligned}$$

$$\begin{aligned} \lim_{t \rightarrow 2^+} \frac{t^2 + 3t - 10}{t - 2} &= \lim_{t \rightarrow 2^+} \frac{t^2 + 5t - 2t - 10}{t - 2} = \lim_{t \rightarrow 2^+} \frac{t(t+5) - 2(t+5)}{t - 2} \\ &= \lim_{t \rightarrow 2^+} \frac{(t+5)(t-2)}{t-2} = \lim_{t \rightarrow 2^+} (t+5) = 2+5 = 7 \end{aligned}$$

Thus,

$$\lim_{t \rightarrow 2} \frac{t^2 + 3t - 10}{t - 2} = 7$$

To be continuous at $t = 2$ we must define $h(2) = 7$.

$$41. f(s) = \frac{s^3 - 1}{s^2 - 1}$$

Find one-sided limits at $s = 1$:

$$\lim_{s \rightarrow 1^-} \frac{s^3 - 1}{s^2 - 1} = \lim_{s \rightarrow 1^-} \frac{(s-1)(s^2 + s + 1)}{(s-1)(s+1)} = \lim_{s \rightarrow 1^-} \frac{s^2 + s + 1}{s+1} = \frac{1^2 + 1 + 1}{1+1} = \frac{3}{2}$$

$$\lim_{s \rightarrow 1^+} \frac{s^3 - 1}{s^2 - 1} = \lim_{s \rightarrow 1^+} \frac{(s-1)(s^2 + s + 1)}{(s-1)(s+1)} = \lim_{s \rightarrow 1^+} \frac{s^2 + s + 1}{s+1} = \frac{1^2 + 1 + 1}{1+1} = \frac{3}{2}$$

Thus,

$$\lim_{s \rightarrow 1} \frac{s^3 - 1}{s^2 - 1} = \frac{3}{2}$$

To be continuous at $s = 1$ we must define $f(1) = \frac{3}{2}$.

$$55. x^3 - 15x + 1 = 0$$

Consider the function $f(x) = x^3 - 15x + 1$.

$$f(-4) = (-4)^3 - 15(-4) + 1 = -3$$

$$f(-3) = (-3)^3 - 15(-3) + 1 = 19$$

$$f(-2) = (-2)^3 - 15(-2) + 1 = 23$$

$$f(-1) = (-1)^3 - 15(-1) + 1 = 15$$

$$f(0) = 0^3 - 15(0) + 1 = 1$$

$$f(1) = 1^3 - 15(1) + 1 = -13$$

$$f(2) = 2^3 - 15(2) + 1 = -21$$

$$f(3) = 3^3 - 15(3) + 1 = -17$$

$$f(4) = 4^3 - 15(4) + 1 = 5$$

f is a polynomial function, so it is continuous everywhere. f changes its sign on the intervals $(-4, -3)$, $(0, 1)$ and $(3, 4)$. By the Intermediate Value Theorem f must have three zeros on these intervals.

$$56. F(x) = (x-a)^2(x-b)^2 + x$$

$$F(a) = (a-a)^2(a-b)^2 + a = 0 + a = a$$

$$F(b) = (b-a)^2(b-b)^2 + b = 0 + b = b$$

The number $\frac{a+b}{2}$ lies between a and b , so by the Intermediate Value Theorem there exists a number c between a and b such that $f(c) = \frac{a+b}{2}$.

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$$16. f(x) = \frac{3x+7}{x^2-2}$$

$$(a) \lim_{x \rightarrow \infty} \frac{3x+7}{x^2-2} = \lim_{x \rightarrow \infty} \frac{\frac{3x}{x^2} + \frac{7}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x^2}} = \lim_{x \rightarrow \infty} \frac{\frac{3}{x} + \frac{7}{x^2}}{1 - \frac{2}{x^2}} = \frac{0+0}{1-0} = 0$$

$$(b) \lim_{x \rightarrow -\infty} \frac{3x+7}{x^2-2} = \lim_{x \rightarrow -\infty} \frac{\frac{3x}{x^2} + \frac{7}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x^2}} = \lim_{x \rightarrow -\infty} \frac{\frac{3}{x} + \frac{7}{x^2}}{1 - \frac{2}{x^2}} = \frac{0+0}{1-0} = 0$$

$$17. h(x) = \frac{7x^3}{x^3-3x^2+6x}$$

$$(a) \lim_{x \rightarrow \infty} \frac{7x^3}{x^3-3x^2+6x} = \lim_{x \rightarrow \infty} \frac{\frac{7x^3}{x^3}}{\frac{x^3}{x^3} - \frac{3x^2}{x^3} + \frac{6x}{x^3}} = \lim_{x \rightarrow \infty} \frac{7}{1 - \frac{3}{x} + \frac{6}{x^2}} = \frac{7}{1-0+0} = 7$$

$$(b) \lim_{x \rightarrow -\infty} \frac{7x^3}{x^3 - 3x^2 + 6x} = \lim_{x \rightarrow -\infty} \frac{\frac{7x^3}{x^3}}{\frac{x^3}{x^3} - \frac{3x^2}{x^3} + \frac{6x}{x^3}} = \lim_{x \rightarrow -\infty} \frac{7}{1 - \frac{3}{x} + \frac{6}{x^2}} = \frac{7}{1 - 0 + 0} = 7$$

$$19. g(x) = \frac{10x^5 + x^4 + 31}{x^6}$$

$$(a) \lim_{x \rightarrow \infty} \frac{10x^5 + x^4 + 31}{x^6} = \lim_{x \rightarrow \infty} \frac{\frac{10x^5}{x^6} + \frac{x^4}{x^6} + \frac{31}{x^6}}{\frac{x^6}{x^6}} = \lim_{x \rightarrow \infty} \frac{\frac{10}{x} + \frac{1}{x^2} + \frac{31}{x^6}}{1} = \frac{0 + 0 + 0}{1} = 0$$

$$(b) \lim_{x \rightarrow -\infty} \frac{10x^5 + x^4 + 31}{x^6} = \lim_{x \rightarrow -\infty} \frac{\frac{10x^5}{x^6} + \frac{x^4}{x^6} + \frac{31}{x^6}}{\frac{x^6}{x^6}} = \lim_{x \rightarrow -\infty} \frac{\frac{10}{x} + \frac{1}{x^2} + \frac{31}{x^6}}{1} = \frac{0 + 0 + 0}{1} = 0$$

$$20. h(x) = \frac{9x^4 + x}{2x^4 + 5x^2 - x + 6}$$

(a)

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{9x^4 + x}{2x^4 + 5x^2 - x + 6} &= \lim_{x \rightarrow \infty} \frac{\frac{9x^4}{x^4} + \frac{x}{x^4}}{\frac{2x^4}{x^4} + \frac{5x^2}{x^4} - \frac{x}{x^4} + \frac{6}{x^4}} = \lim_{x \rightarrow \infty} \frac{9 + \frac{1}{x^3}}{2 + \frac{5}{x^2} - \frac{1}{x^3} + \frac{6}{x^4}} \\ &= \frac{9 + 0}{2 + 0 - 0 + 0} = \frac{9}{2} \end{aligned}$$

(b)

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{9x^4 + x}{2x^4 + 5x^2 - x + 6} &= \lim_{x \rightarrow -\infty} \frac{\frac{9x^4}{x^4} + \frac{x}{x^4}}{\frac{2x^4}{x^4} + \frac{5x^2}{x^4} - \frac{x}{x^4} + \frac{6}{x^4}} = \lim_{x \rightarrow -\infty} \frac{9 + \frac{1}{x^3}}{2 + \frac{5}{x^2} - \frac{1}{x^3} + \frac{6}{x^4}} \\ &= \frac{9 + 0}{2 + 0 - 0 + 0} = \frac{9}{2} \end{aligned}$$

$$22. h(x) = \frac{-x^4}{x^4 - 7x^3 + 7x^2 + 9}$$

$$(a) \lim_{x \rightarrow \infty} \frac{-x^4}{x^4 - 7x^3 + 7x^2 + 9} = \lim_{x \rightarrow \infty} \frac{\frac{-x^4}{x^4}}{\frac{x^4}{x^4} - \frac{7x^3}{x^4} + \frac{7x^2}{x^4} + \frac{9}{x^4}} = \lim_{x \rightarrow \infty} \frac{-1}{1 - \frac{7}{x} + \frac{7}{x^2} + \frac{9}{x^4}} = \frac{-1}{1} = -1$$

$$(b) \lim_{x \rightarrow -\infty} \frac{-x^4}{x^4 - 7x^3 + 7x^2 + 9} = \lim_{x \rightarrow -\infty} \frac{\frac{-x^4}{x^4}}{\frac{x^4}{x^4} - \frac{7x^3}{x^4} + \frac{7x^2}{x^4} + \frac{9}{x^4}} = \lim_{x \rightarrow -\infty} \frac{-1}{1 - \frac{7}{x} + \frac{7}{x^2} + \frac{9}{x^4}} = \frac{-1}{1} = -1$$

24.

$$\begin{aligned} \lim_{x \rightarrow -\infty} \left(\frac{x^2 + x - 1}{8x^2 - 3} \right)^{\frac{1}{3}} &= \lim_{x \rightarrow -\infty} \left(\frac{\frac{x^2}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}}{\frac{8x^2}{x^2} - \frac{3}{x^2}} \right)^{\frac{1}{3}} = \lim_{x \rightarrow -\infty} \left(\frac{1 + \frac{1}{x} - \frac{1}{x^2}}{8 - \frac{3}{x^2}} \right)^{\frac{1}{3}} \\ &= \left(\frac{1+0-0}{8-0} \right)^{\frac{1}{3}} = \left(\frac{1}{8} \right)^{\frac{1}{3}} = \frac{1}{2} \end{aligned}$$

$$29. \lim_{x \rightarrow \infty} \frac{\sqrt[3]{x} - \sqrt[5]{x}}{\sqrt[3]{x} + \sqrt[5]{x}} = \lim_{x \rightarrow \infty} \frac{\frac{x^{\frac{1}{3}} - x^{\frac{1}{5}}}{x^{\frac{1}{3}} + x^{\frac{1}{5}}}}{\frac{\frac{1}{x^{\frac{1}{3}}} - \frac{1}{x^{\frac{1}{5}}}}{\frac{1}{x^{\frac{1}{3}}} + \frac{1}{x^{\frac{1}{5}}}}} = \lim_{x \rightarrow \infty} \frac{\frac{1 - \frac{1}{x^{\frac{2}{15}}}}{1 + \frac{1}{x^{\frac{2}{15}}}}}{\frac{1 - \frac{1}{x^{\frac{2}{15}}}}{1 + \frac{1}{x^{\frac{2}{15}}}}} = \frac{1-0}{1+0} = 1$$

$$30. \lim_{x \rightarrow \infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}} = \lim_{x \rightarrow \infty} \frac{\frac{x^{-1} - x^{-4}}{x^{-2} - x^{-3}}}{\frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}} = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^{-1+4}}}{\frac{1}{x^{-1+2}} + \frac{1}{x^{-1+3}}} = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^3}}{\frac{1}{x} + \frac{1}{x^2}} = \infty$$

$$43. \lim_{x \rightarrow 7} \frac{4}{(x-7)^2} = \infty$$

because $(x-7)^2 \rightarrow 0$ as $x \rightarrow 7$.

$$44. \lim_{x \rightarrow 0} \frac{-1}{x^2(x+1)} = -\infty$$

because $x^2 \rightarrow 0$ and $x+1 \rightarrow 1$ as $x \rightarrow 0$.

$$55. \text{ (a) } \lim_{x \rightarrow 0^+} \left(\frac{x^2}{2} - \frac{1}{x} \right) = \lim_{x \rightarrow 0^+} \frac{x^2}{2} - \lim_{x \rightarrow 0^+} \frac{1}{x} = 0 - \lim_{x \rightarrow 0^+} \frac{1}{x} = -\infty$$

$$\text{ (b) } \lim_{x \rightarrow 0^-} \left(\frac{x^2}{2} - \frac{1}{x} \right) = \lim_{x \rightarrow 0^-} \frac{x^2}{2} - \lim_{x \rightarrow 0^-} \frac{1}{x} = 0 - \lim_{x \rightarrow 0^-} \frac{1}{x} = \infty$$

$$\text{ (c) } \lim_{x \rightarrow \sqrt[3]{2}} \left(\frac{x^2}{2} - \frac{1}{x} \right) = \frac{(\sqrt[3]{2})^2}{2} - \frac{1}{\sqrt[3]{2}} = \frac{\sqrt[3]{4}}{2} - \frac{1}{\sqrt[3]{2}} = \frac{\sqrt[3]{8} - 2}{2\sqrt[3]{2}} = \frac{2 - 2}{2\sqrt[3]{2}} = 0$$

$$\text{ (d) } \lim_{x \rightarrow -1} \left(\frac{x^2}{2} - \frac{1}{x} \right) = \frac{(-1)^2}{2} - \frac{1}{-1} = \frac{1}{2} + 1 = \frac{3}{2}$$

56.

$$\text{ (a) } \lim_{x \rightarrow -2^+} \frac{x^2 - 1}{2x + 4} = \lim_{x \rightarrow -2^+} \frac{x^2 - 1}{2(x + 2)} = \infty$$

because $x^2 - 1 \rightarrow 3$ and $\frac{1}{x + 2} \rightarrow \infty$ as $x \rightarrow -2^+$.

$$\text{ (b) } \lim_{x \rightarrow -2^-} \frac{x^2 - 1}{2x + 4} = \lim_{x \rightarrow -2^-} \frac{x^2 - 1}{2(x + 2)} = -\infty$$

because $x^2 - 1 \rightarrow 3$ and $\frac{1}{x + 2} \rightarrow -\infty$ as $x \rightarrow -2^-$.

$$\text{ (c) } \lim_{x \rightarrow 1^+} \frac{x^2 - 1}{2x + 4} = \lim_{x \rightarrow 1^+} \frac{(x - 1)(x + 1)}{2x + 4} = 0$$

because $x - 1 \rightarrow 0$, $x + 1 \rightarrow 2$ and $2x + 4 \rightarrow 6$ as $x \rightarrow 1^+$.

$$\text{ (d) } \lim_{x \rightarrow 0^-} \frac{x^2 - 1}{2x + 4} = \frac{0^2 - 1}{2 \cdot 0 + 4} = -\frac{1}{4}$$

80.

$$\lim_{x \rightarrow \infty} (\sqrt{x+9} - \sqrt{x+4}) = \lim_{x \rightarrow \infty} \frac{(\sqrt{x+9} - \sqrt{x+4})(\sqrt{x+9} + \sqrt{x+4})}{\sqrt{x+9} + \sqrt{x+4}}$$

$$= \lim_{x \rightarrow \infty} \frac{x+9 - (x+4)}{\sqrt{x+9} + \sqrt{x+4}} = \lim_{x \rightarrow \infty} \frac{x+9 - x - 4}{\sqrt{x+9} + \sqrt{x+4}}$$

$$= \lim_{x \rightarrow \infty} \frac{5}{\sqrt{x+9} + \sqrt{x+4}} = 0$$

because $\sqrt{x+9} + \sqrt{x+4} \rightarrow \infty$ as $x \rightarrow \infty$.

81.

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 + 25} - \sqrt{x^2 - 1}) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 25} - \sqrt{x^2 - 1})(\sqrt{x^2 + 25} + \sqrt{x^2 - 1})}{\sqrt{x^2 + 25} + \sqrt{x^2 - 1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 + 25 - (x^2 - 1)}{\sqrt{x^2 + 25} + \sqrt{x^2 - 1}} = \lim_{x \rightarrow \infty} \frac{x^2 + 25 - x^2 + 1}{\sqrt{x^2 + 25} + \sqrt{x^2 - 1}} \\ &= \lim_{x \rightarrow \infty} \frac{26}{\sqrt{x^2 + 25} + \sqrt{x^2 - 1}} = 0 \\ &\text{because } \sqrt{x^2 + 25} + \sqrt{x^2 - 1} \rightarrow \infty \text{ as } x \rightarrow \infty. \end{aligned}$$

101. $y = \frac{x^2 - 4}{x - 1}$

The domain is the intervals $(-\infty, 1) \cup (1, \infty)$.

$$y(-x) = \frac{(-x)^2 - 4}{-x - 1} = -\frac{x^2 - 4}{x + 1}$$

The function is neither odd nor even.

Find the x-intercepts:

$$\frac{x^2 - 4}{x - 1} = 0$$

$$x^2 - 4 = 0$$

$$(x - 2)(x + 2) = 0$$

$$x = 2 \text{ or } x = -2$$

The x-intercepts are $(-2, 0)$ and $(2, 0)$.

Find the y-intercept:

$$y(0) = \frac{0^2 - 4}{0 - 1} = 4$$

The y-intercept is $(0, 4)$.

The denominator is zero at $x = -1$, so the line $x = -1$ is a vertical asymptote.

To find the equation of a slant asymptote we use the long division:

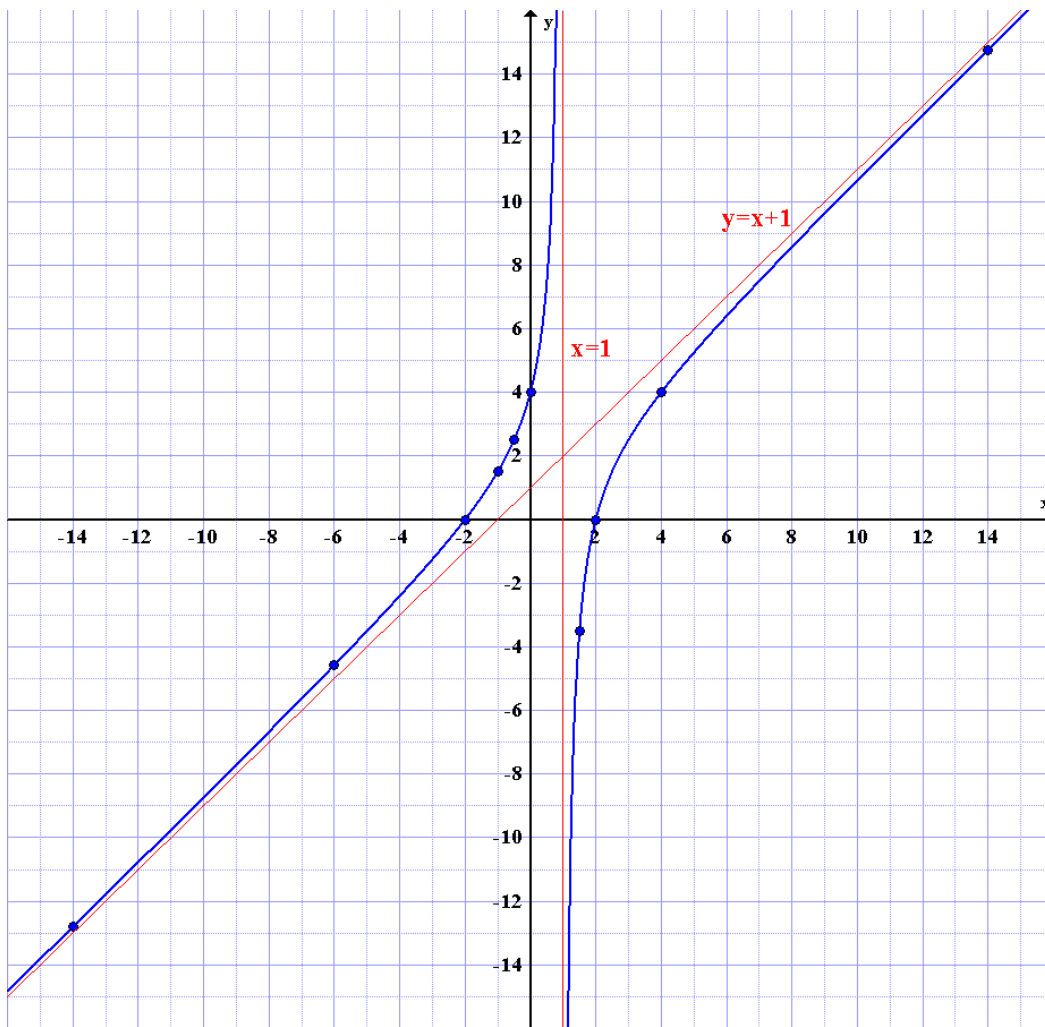
$$\begin{array}{r} x+1 \\ x-1 \overline{) x^2 - 4} \\ \underline{x^2 - x} \\ x - 4 \\ \underline{x - 1} \\ -3 \end{array}$$

Thus, $\frac{x^2 - 4}{x - 1} = x + 1 - \frac{3}{x - 1}$

The slant asymptote is $y = x + 1$

Find few points:

x	-14	-6	-1	$-\frac{1}{2}$	$\frac{3}{2}$	4	14
$y = \frac{x^2 - 4}{x - 1}$	-12.8	-4.57	1.5	2.5	-3.5	4	14.77



$$102. y = \frac{x^2 - 1}{2x + 4}$$

The domain is the intervals $(-\infty, -2) \cup (-2, \infty)$.

$$y(-x) = \frac{(-x)^2 - 1}{2(-x) + 4} = \frac{x^2 - 1}{-2x + 4}$$

The function is neither odd nor even.

Find the x-intercepts:

$$\frac{x^2 - 1}{2x + 4} = 0$$

$$x^2 - 1 = 0$$

$$(x - 1)(x + 1) = 0$$

$$x = 1 \quad \text{or} \quad x = -1$$

The x-intercepts are $(-1, 0)$ and $(1, 0)$.

Find the y-intercept:

$$y(0) = \frac{0^2 - 1}{2(0) + 4} = -\frac{1}{4}$$

The y-intercept is $\left(0, -\frac{1}{4}\right)$.

The denominator is zero at $x = -2$, so the line $x = -2$ is a vertical asymptote.

To find the equation of a slant asymptote we use the long division:

$$\begin{array}{r} \frac{1}{2}x - 1 \\ 2x + 4 \overline{) x^2 - 1} \\ \underline{x^2 + 2x} \\ -2x - 1 \\ \underline{-2x - 4} \\ 3 \end{array}$$

$$\text{Thus, } \frac{x^2 - 1}{2x + 4} = \frac{1}{2}x - 1 + \frac{3}{2x + 4}$$

The slant asymptote is $y = \frac{1}{2}x - 1$.

Find few points:

x	-14	-8	-4	-3	$-\frac{5}{2}$	$-\frac{3}{2}$	4	8	14
$y = \frac{x^2 - 1}{2x + 4}$	-8.13	-5.25	-3.75	-4	-5.25	1.25	1.25	3.15	6.09

