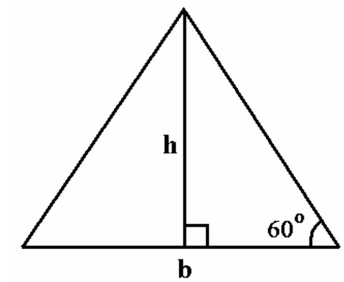
Pg.11

9.



The perimeter is

P = 3x

The area of a triangle is

$$A = \frac{1}{2}bh$$

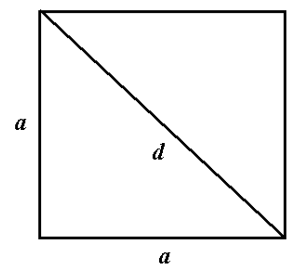
where b is the base, h is the height

$$h = \frac{1}{2}b\tan 60^0 = \frac{1}{2}b \cdot \sqrt{3} = \frac{\sqrt{3}}{2}b$$

In our case b = x, then the area is

$$A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

10. By Pythagorean theorem



$$a^2 + a^2 = d^2$$

$$2a^2 = d^2$$

$$a^2 = \frac{d^2}{2}$$

$$a = \sqrt{\frac{d^2}{2}}$$

$$a = \frac{d}{\sqrt{2}}$$

$$a = \frac{\sqrt{2}d}{2}$$

The area of the square is

$$A = a^2 = \left(\frac{\sqrt{2}d}{2}\right)^2 = \frac{2d^2}{4} = \frac{1}{2}d^2$$

11. From the triangle ABC by Pythagorean theorem

$AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$

From the triangle ACD by Pythagorean theorem

$AD^2 = AC^2 + CD^2 = 2a^2 + a^2 = 3a^2$

Thus,

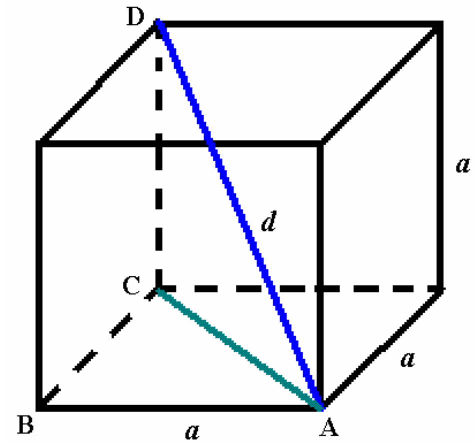
$$d^2 = 3a^2$$

$$a^2 = \frac{d^2}{3}$$

$$a = \sqrt{\frac{d^2}{3}}$$

$$a = \frac{d}{\sqrt{3}}$$

$$a = \frac{\sqrt{3}d}{3}$$



The surface area is

$$A = 6a^2 = 6\left(\frac{\sqrt{3}d}{3}\right)^2 = 6 \cdot \frac{d^2}{3} = 2d^2$$

The volume is

$$V = a^3 = \left(\frac{\sqrt{3}d}{3}\right)^3 = \frac{3\sqrt{3}d^3}{27} = \frac{\sqrt{3}d^3}{9}$$

23.

(a) |y|= x

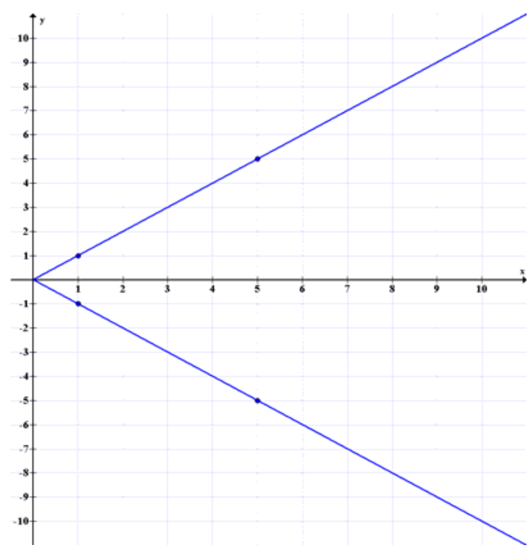
This equation can be rewritten as

$$x = \begin{cases} -y, & y < 0 \\ y, & y \geq 0 \end{cases}$$

The first line passes through the points (1, –1) and (5, –5).

The second line passes through the points (1, 1) and (5, 5).

The graph is not graph of a function because we can draw a vertical line that intersects it more than once.



(b) $y^2 = x^2$

This equation can be rewritten as

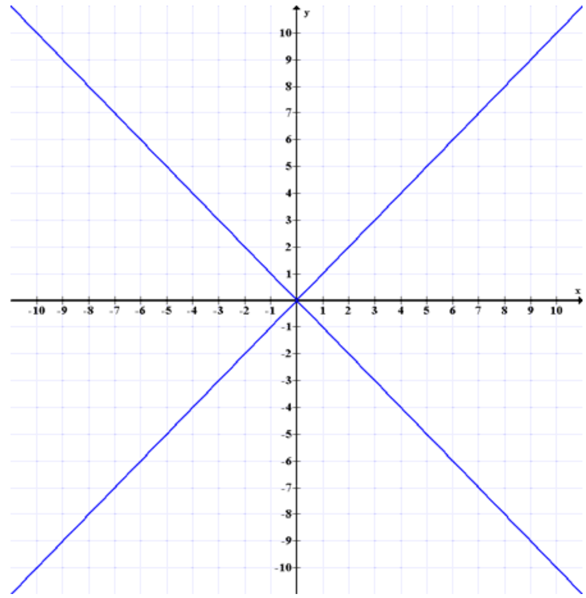
$y^2 - x^2 = 0$

$(y - x)(y + x) = 0$

$y - x = 0$ or $y + x = 0$

$y = x$ or $y = -x$

The graph is not graph of a function because we can draw a vertical line that intersects it more than once.



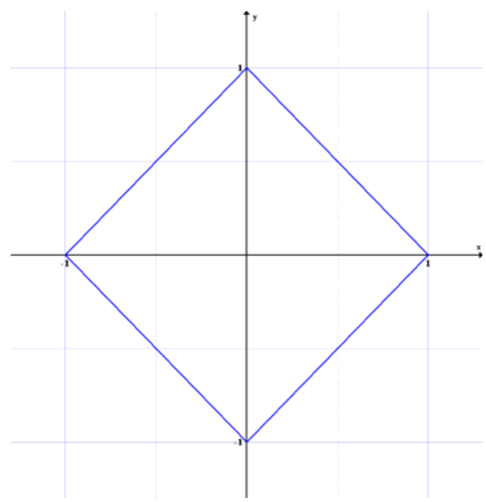
24. (a) $|x| + |y| = 1$

In the Quadrant I $x > 0$ and $y > 0$, so we have

$x + y = 1$

$y = -x + 1$

This is a line with the slope $m = -1$ and the y-intercept (0, 1).



In the Quadrant II $x < 0$ and $y > 0$, so we have

$-x + y = 1$

$y = x + 1$

This is a line with the slope $m = 1$ and the y-intercept (0, 1).

In the Quadrant III $x < 0$ and $y < 0$, so we have

$-x - y = 1$

$y = -x - 1$

This is a line with the slope $m = -1$ and the y-intercept (0, –1).

In the Quadrant IV $x > 0$ and $y < 0$, so we have

$x - y = 1$

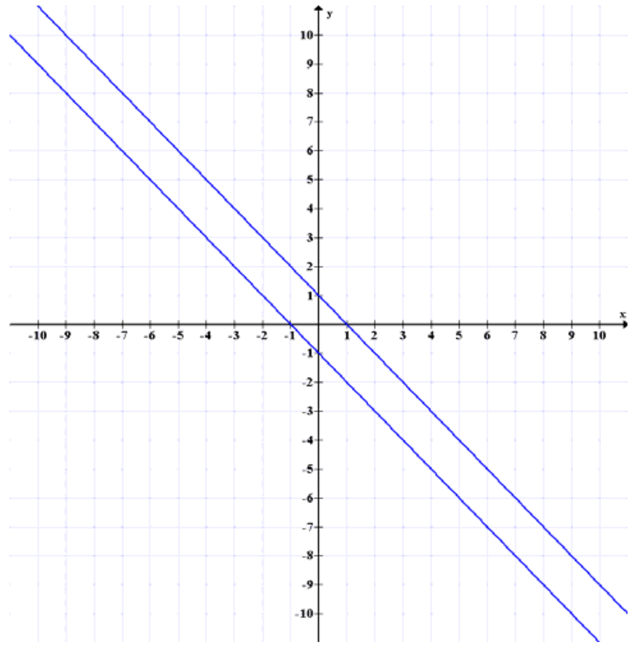
$y = x - 1$

This is a line with the slope $m = 1$ and the y-intercept (0, –1).

The graph is not graph of a function because we can draw a vertical line that intersects it more than once.

(b) $|x + y| = 1$

We have two equations:
$x + y = 1$ and $x + y = -1$
or
$y = -x + 1$ and $y = -x - 1$
The first graph is a line with the slope $m = -1$ and the y-intercept (0, 1).
The second graph is a line with the slope $m = -1$ and the y-intercept (0, –1).
The graph is not graph of a function because we can draw a vertical line that intersects it more than once.



27.

$$F(x) = \begin{cases} 4 - x^2, & x \le 1 \\ x^2 + 2x, & x > 1 \end{cases}$$

When $x \le 1$ then $F(x) = 4 - x^2$, so the part of the graph of F that lies to the left of the vertical line x= 1 must coincide with the parabola $y = 4 - x^2$.
The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{0}{2(-1)} = 0$$

Find y for x = 0:
$y = 4 - 0^2 = 4$.
The vertex is (0, 4).
Find few points:

x	1	–1	–2	–3	–4
$y = 4 - x^2$	3	3	0	–5	–12

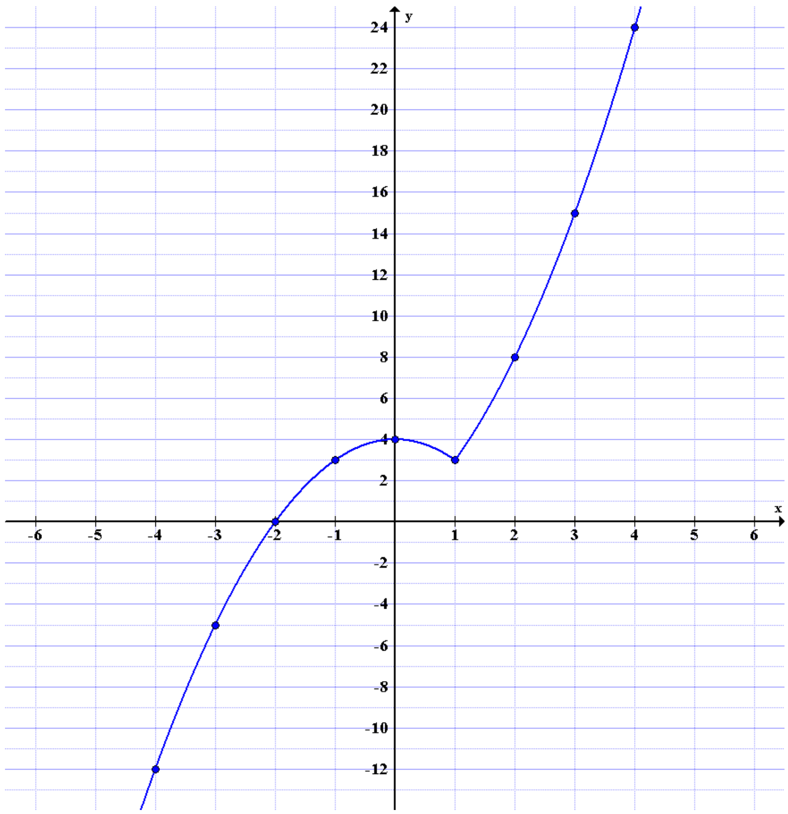
When $x > 1$ then $F(x) = x^2 + 2x$, so the part of the graph of F that lies to the right of the vertical line x= 1 must coincide with the parabola $y = x^2 + 2x$.
The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{-2}{2\cdot 1} = 1$$

Find y for x = 1:
$y = 1^2 + 2(1) = 3$

The vertex is (1, 3).
Find few points:

x	2	3	4
$y = x^2 + 2x$	8	15	24



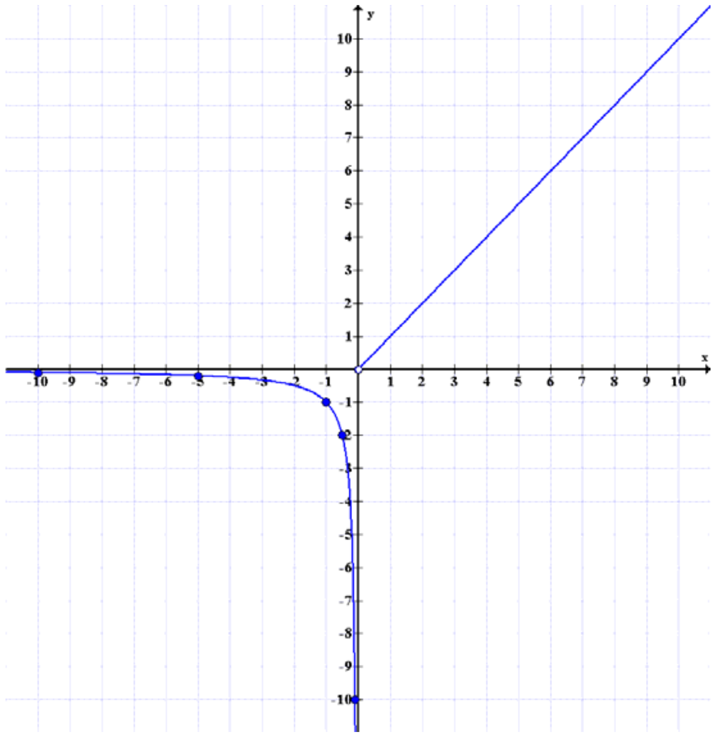
28.

$$G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & x > 0 \end{cases}$$

When x < 0 then G(x) = 1/x, so the part of the graph of F that lies to the left of the vertical line x= 0 must coincide with the graph of y = 1/x.
Find few points:

x	–10	–5	–1	–0.5	–0.1
y = 1/x	–0.1	–0.2	–1	–2	–10

When x > 0 then G(x) = x, so the part of the graph of F that lies to the right of the vertical line x= 0 must coincide with the graph of y = x.
This line have slope m = 1 and y-intercept (0, 0).



29. (a) On the interval [0, 1] the line passes through the point (0, 0) and (1, 1), so the equation of the graph on this interval is y = x.
On the interval [1, 2] the line passes through the point (1, 1) and (2, 0).
The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{2-1} = -1$$

The equation of the line is

$y - y_1 = m(x - x_1)$

$y - 1 = -(x - 1)$

$y - 1 = -x + 1$

$y = -x + 2$

The equation of the function is

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ -x+2, & 1 \le x \le 2 \end{cases}$$

(b) The equation of the function is

$$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$$

30. (a) On the interval (0, 2] the line passes through the point (0, 2) and (2, 0).

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{2-0} = -1$$

The equation of the line is

$y - y_1 = m(x - x_1)$

$y - 2 = -(x - 0)$

$y = -x + 2$

On the interval (2, 5] the line passes through the point (2, 1) and (5, 0).

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{5-2} = -\frac{1}{3}$$

The equation of the line is

$y - y_1 = m(x - x_1)$

$$y - 0 = -\frac{1}{3}(x - 5)$$

$$y = -\frac{1}{3}x + \frac{5}{3}$$

The equation of the function is

$$f(x) = \begin{cases} -x + 2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

(b) On the interval (–1, 0] the line passes through the point (–1, 0) and (0, –3).

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{0-(-1)} = -3$$

The equation of the line is

$y - y_1 = m(x - x_1)$

$y - 0 = -3(x - (-1))$

$y = -3x - 3$

On the interval (0, 2] the line passes through the point (0, 3) and (2, –1).

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{2-0} = -2$$

The equation of the line is

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - 0)$

$y = -2x + 3$

The equation of the function is

$$f(x) = \begin{cases} -3x - 3, & -1 < x \le 0 \\ -2x + 3, & 0 < x \le 2 \end{cases}$$

11. f(x) = x – 3, $g(x) = \sqrt{x}$, h(x) = x^3, j(x) = 2x

(a)

$$y = \sqrt{x} - 3$$
$$y = (f \circ g)(x)$$

(b)

$$y = 2\sqrt{x}$$
$$y = (j \circ g)(x)$$

(c)

$$y = x^{\frac{1}{4}}$$
$$y = (g \circ g)(x)$$

(d)

$$y = 4x$$
$$y = (j \circ j)(x)$$

(e)

$$y = \sqrt{(x-3)^3}$$
$$(h \circ f)(x) = (x-3)^3$$
$$y = (g \circ h \circ f)(x)$$

(f)

$$y = (2x-6)^3$$
$$(j \circ f)(x) = (2x-6)$$
$$y = (h \circ j \circ f)(x)$$

12. (a)

$$y = 2x - 3$$
$$y = (f \circ j)(x)$$

(b)

$$y = x^{\frac{3}{2}}$$
$$y = (h \circ g)(x)$$

(c)

$$y = x^9$$
$$y = (h \circ h)(x)$$

(d)

$$y = x - 6$$
$$\frac{1}{4}h(x) = \frac{1}{2}x$$
$$2f(x) = 2x - 6$$
$$y = (2f \circ \frac{1}{4}h)(x)$$

(e)

$$y = 2\sqrt{x-3}$$
$$(g \circ f)(x) = \sqrt{x-3}$$
$$y = (h \circ g \circ f)(x)$$

(f)

$$y = \sqrt{x^3 - 3}$$
$$(f \circ h)(x) = x^3 - 3$$
$$y = (g \circ f \circ h)(x)$$

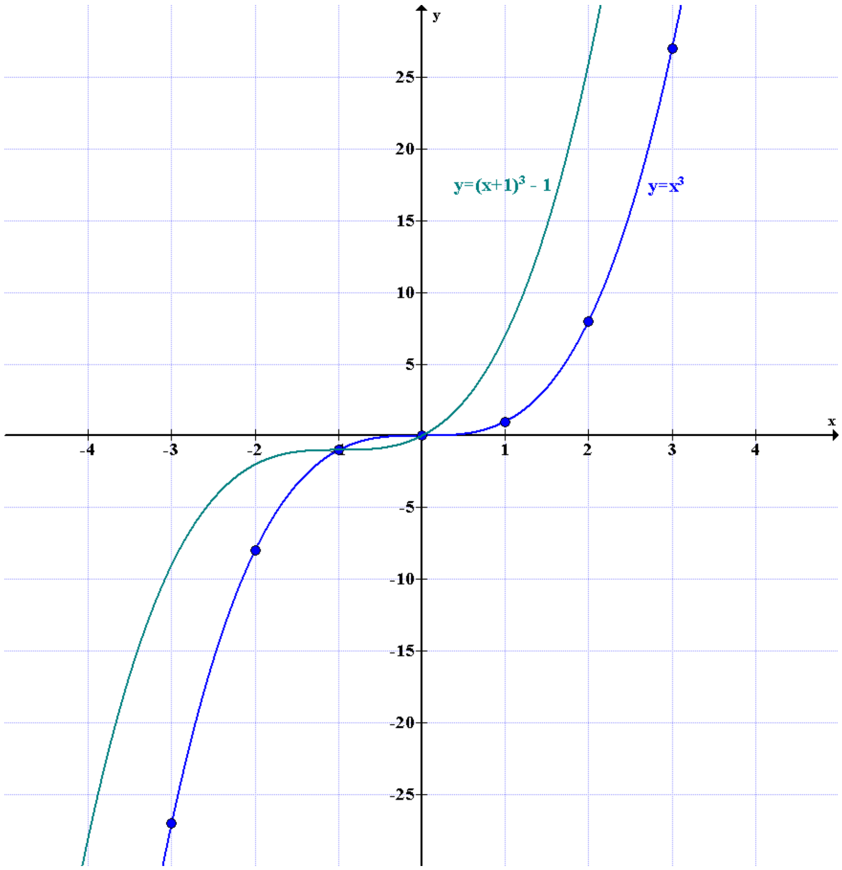
27. $y = x^3$, left 1, down 1

If c > 0 then
y = f(x + c) shifts the graph of y = f(x) a distance c units to the left
y = f(x) – c shifts the graph of y = f(x) a distance c units downward

Therefore,
$y = (x + 1)^3$ shifts the graph of $y = x^3$ one unit to the left
$y = (x + 1)^3 - 1$ shifts the graph of $y = (x + 1)^3$ one unit downward

Find few points:

x	–3	–2	–1	0	1	2	3
$y = x^3$	–27	–8	–1	0	1	8	27



28. $y = x^{\frac{2}{3}}$, right 1, down 1

If c > 0 then
y = f(x – c) shifts the graph of y = f(x) a distance c units to the left
y = f(x) – c shifts the graph of y = f(x) a distance c units downward

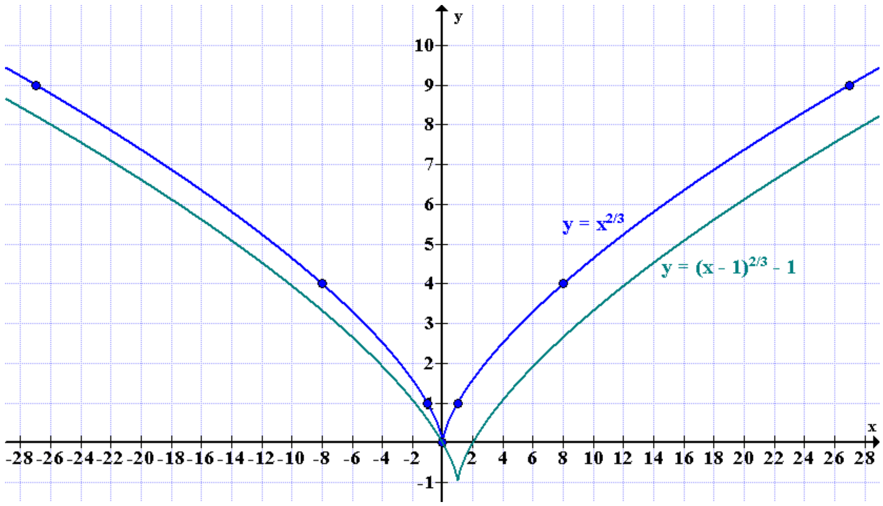
Therefore,

$y = (x-1)^{\frac{2}{3}}$ shifts the graph of $y = x^{\frac{2}{3}}$ one unit to the right

$y = (x-1)^{\frac{2}{3}} - 1$ shifts the graph of $y = (x-1)^{\frac{2}{3}}$ one unit downward

Find few points:

x	–27	–8	–1	0	1	8	27
$y = x^{\frac{2}{3}}$	9	4	1	0	1	4	9

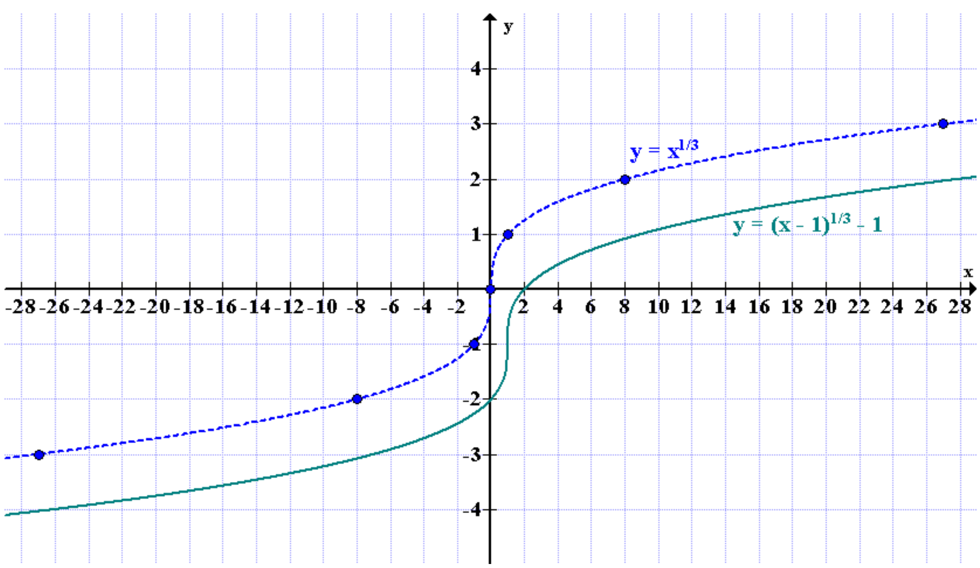


45. $y = \sqrt[3]{x-1} - 1$

The graph of $y = \sqrt[3]{x-1} - 1$ is the graph of $y = \sqrt[3]{x}$ shifted by 1 unit to the right and then moved by 1 unit downward.

Find few points:

x	–27	–8	–1	0	1	8	27
$y = \sqrt[3]{x}$	–3	–2	–1	0	1	2	3

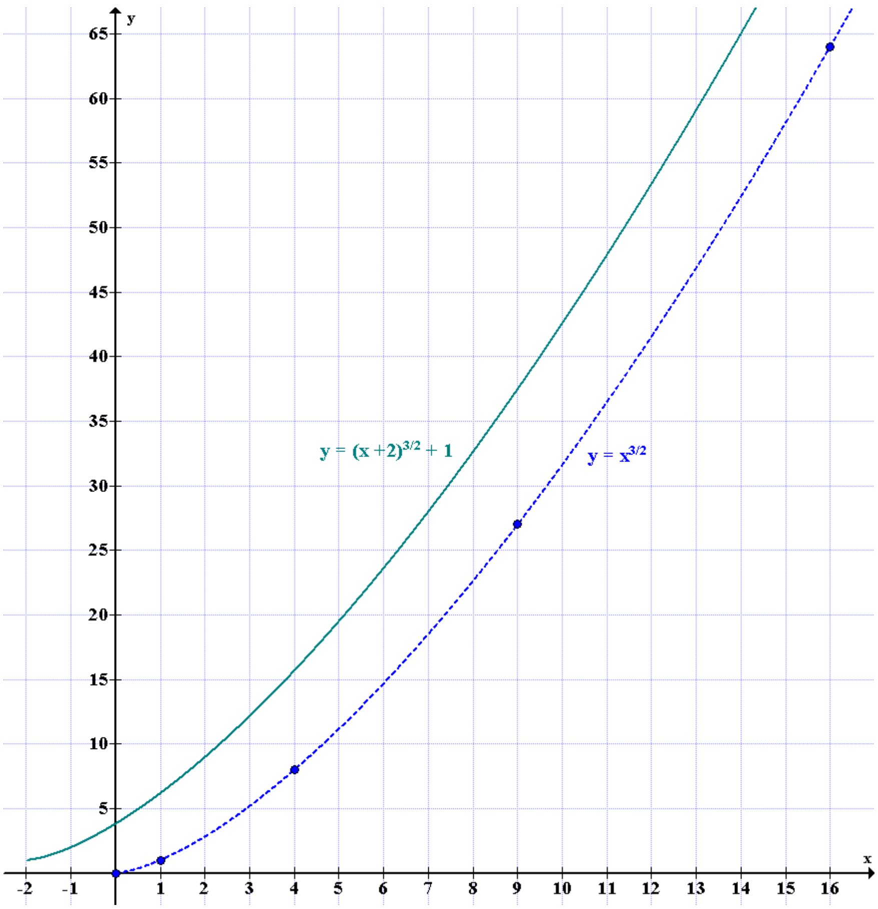


46. $y = (x + 2)^{3/2} + 1$

The graph of $y = (x + 2)^{3/2} + 1$ is the graph of $y = x^{3/2}$ shifted by 2 unit to the left and then moved by 1 unit upward.

Find few points:

x	0	1	4	9	16
$y = x^{3/2}$	0	1	8	27	64



59. $y = 1 + \frac{1}{x^2}$, compressed vertically by a factor of 2

If c > 1 then

$y = \frac{1}{c} f(x)$ compress the graph of $y = f(x)$ vertically by a factor of c.

Then

$$y = \frac{1}{2}\left(1 + \frac{1}{x^2}\right)$$

$$y = \frac{1}{2} + \frac{1}{2x^2}$$

60. $y = 1 + \frac{1}{x^2}$, stretched horizontally by a factor of 3

If c > 1 then

$y = f\left(\frac{x}{c}\right)$ stretch the graph of $y = f(x)$ horizontally by a factor of c.

Then

$$y = 1 + \frac{1}{\left(\frac{x}{3}\right)^2}$$

$$y = 1 + \frac{1}{\left(\frac{x^2}{9}\right)}$$

$$y = 1 + \frac{9}{x^2}$$

Pg.28

9. $\cos x = \frac{1}{3},\ x \in \left[-\frac{\pi}{2}, 0\right]$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin x = \pm\sqrt{1 - \cos^2 x}$$

Since x is in Quadrant IV then

$$\sin x = -\sqrt{1 - \cos^2 x}$$

$$\sin x = -\sqrt{1 - \left(\frac{1}{3}\right)^2} = -\sqrt{1 - \frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{\sqrt{8}}{3} = -\frac{2\sqrt{2}}{3}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = -2\sqrt{2}$$

10. $\cos x = -\frac{5}{13}$, $x \in \left[\frac{\pi}{2}, \pi\right]$

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$
$$\sin x = \pm\sqrt{1-\cos^2 x}$$

Since x is in Quadrant II then

$$\sin x = \sqrt{1-\cos^2 x}$$
$$\sin x = \sqrt{1-\left(-\frac{5}{13}\right)^2} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$
$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{12}{13}}{-\frac{5}{13}} = -\frac{12}{5}$$

11. $\tan x = \frac{1}{2}$, $x \in \left[\pi, \frac{3\pi}{2}\right]$

$$\sin^2 x + \cos^2 x = 1$$
$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$$
$$\tan^2 x + 1 = \frac{1}{\cos^2 x}$$
$$\cos^2 x = \frac{1}{\tan^2 x + 1}$$
$$\cos x = \pm\sqrt{\frac{1}{\tan^2 x + 1}}$$
$$\cos x = \pm\frac{1}{\sqrt{\tan^2 x + 1}}$$

Since x is in Quadrant III then

$$\cos x = -\frac{1}{\sqrt{\tan^2 x + 1}}$$

$$\cos x = -\frac{1}{\sqrt{\left(\frac{1}{2}\right)^2+1}} = -\frac{1}{\sqrt{\frac{1}{4}+1}} = -\frac{1}{\sqrt{\frac{5}{4}}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\sin x = \tan x \cos x = \frac{1}{2}\left(-\frac{2\sqrt{5}}{5}\right) = -\frac{\sqrt{5}}{5}$$

19. $y = \cos\left(x - \frac{\pi}{2}\right)$

The graph of $y = \cos\left(x - \frac{\pi}{2}\right)$ is the graph of $y = \cos x$ shifted by $\frac{\pi}{2}$ units to the right.

Find the y-intercept:

$y = \cos\left(0 - \frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

The y-intercept is (0, 0).

Find the x-intercepts:

$\cos\left(x - \frac{\pi}{2}\right) = 0$

$x - \frac{\pi}{2} = \frac{\pi}{2} + \pi k \qquad k \in Z$

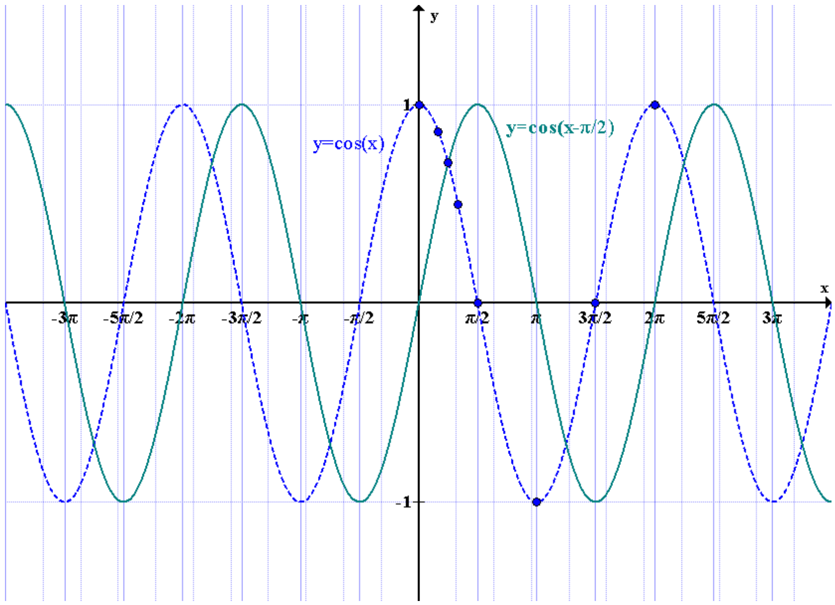
$x = \pi + \pi k$

The x-intercepts are $(\pi + \pi k, 0)$.

Find few points:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y = cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1

y = cos x is an even function so its graph is symmetric about the y-axis.



The period is 2π.

20. $y = \sin\left(x + \frac{\pi}{6}\right)$

The graph of $y = \sin\left(x + \frac{\pi}{6}\right)$ is the graph of $y = \sin x$ shifted by $\frac{\pi}{6}$ units to the left.

Find the y-intercept:

$$y = \sin\left(0 + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

The y-intercept is $\left(0, \frac{1}{2}\right)$.

Find the x-intercepts:

$$\sin\left(x + \frac{\pi}{6}\right) = 0$$

$$x + \frac{\pi}{6} = \pi k \qquad k \in Z$$

$$x = -\frac{\pi}{6} + \pi k$$

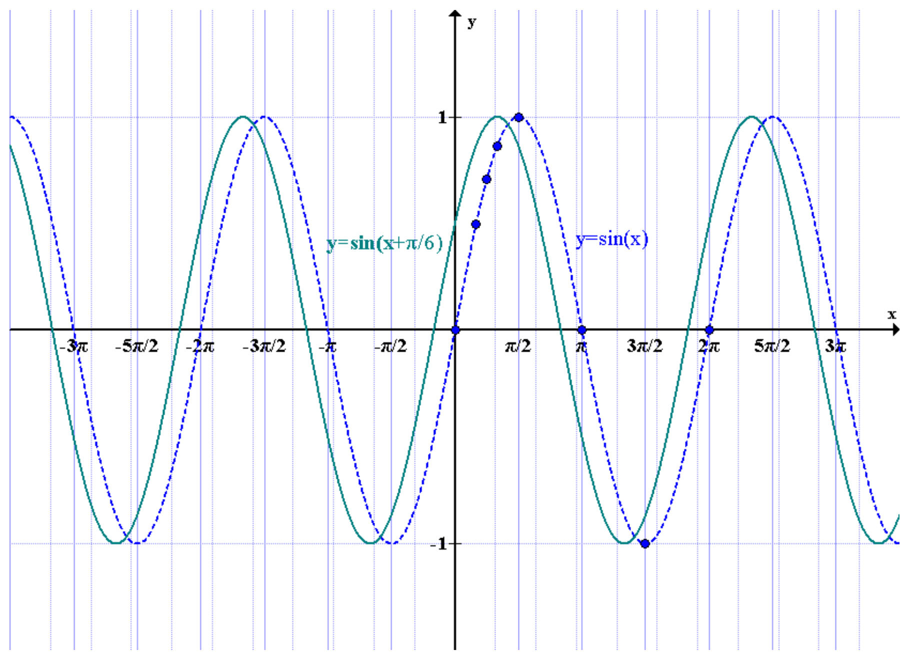
The x-intercepts are $\left(-\frac{\pi}{6} + \pi k, 0\right)$.

Find few points:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

y = sin x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0

y = sin x is an odd function so its graph is symmetric about the origin.



The period is 2π.

31. $\cos\left(x-\frac{\pi}{2}\right)=\sin x$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

Transform the left side:

$\cos\left(x-\frac{\pi}{2}\right)=\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}$

$=\cos x\cdot 0+\sin x\cdot 1$

$=\sin x$

32. $\cos\left(x+\frac{\pi}{2}\right)=-\sin x$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

Transform the left side:

$$\cos\left(x+\frac{\pi}{2}\right)=\cos x\cos\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}$$

$$=\cos x\cdot 0-\sin x\cdot 1$$

$$=-\sin x$$

47. $\cos^2\frac{\pi}{8}$

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2}$$

$$\cos^2\frac{\pi}{8}=\frac{1+\cos\frac{\pi}{4}}{2}=\frac{1+\frac{\sqrt{2}}{2}}{2}=\frac{2+\sqrt{2}}{4}$$

48. $\cos^2\frac{\pi}{8}$

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2}$$

$$\cos^2\frac{5\pi}{12}=\frac{1+\cos\frac{5\pi}{6}}{2}=\frac{1+\cos\left(\pi-\frac{\pi}{6}\right)}{2}=\frac{1+\cos\pi\cos\frac{\pi}{6}+\sin\pi\sin\frac{\pi}{6}}{2}$$

$$=\frac{1+(-1)\cdot\frac{\sqrt{3}}{2}+0\cdot\frac{1}{2}}{2}=\frac{1-\frac{\sqrt{3}}{2}}{2}$$

$$=\frac{2-\sqrt{3}}{4}$$

53. $\sin 2\theta-\cos\theta=0$

$$\sin 2\theta - \cos\theta = 0$$
$$2\sin\theta\cos\theta - \cos\theta = 0$$
$$\cos\theta(2\sin\theta - 1) = 0$$
$$\cos\theta = 0 \quad or \quad 2\sin\theta - 1 = 0$$
$$\cos\theta = 0 \quad or \quad \sin\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{2} + \pi k \quad k \in Z \quad or \quad \theta = (-1)^m \sin^{-1}\frac{1}{2} + m\pi \quad m \in Z$$
$$\theta = \frac{\pi}{2} + \pi k \quad or \quad \theta = (-1)^m \frac{\pi}{6} + \pi m$$

54. $\cos 2\theta + \cos\theta = 0$

$$\cos^2\theta - \sin^2\theta + \cos\theta = 0$$
$$\cos^2\theta - (1 - \cos^2\theta) + \cos\theta = 0$$
$$\cos^2\theta - 1 + \cos^2\theta + \cos\theta = 0$$
$$2\cos^2\theta - 1 + \cos\theta = 0$$
$$2\cos^2\theta + \cos\theta - 1 = 0$$
$$2\cos^2\theta + 2\cos\theta - \cos\theta - 1 = 0$$
$$2\cos\theta(\cos\theta + 1) - (\cos\theta + 1) = 0$$
$$(\cos\theta + 1)(2\cos\theta - 1) = 0$$
$$\cos\theta + 1 = 0 \quad or \quad 2\cos\theta - 1 = 0$$
$$\cos\theta = -1 \quad or \quad 2\cos\theta = 1$$
$$\cos\theta = -1 \quad or \quad \cos\theta = \frac{1}{2}$$
$$\theta = \pi + 2\pi k \quad k \in Z \quad or \quad \theta = \pm\cos^{-1}\frac{1}{2} + 2\pi m \quad m \in Z$$
$$\theta = \pi + 2\pi k \quad or \quad \theta = \pm\frac{\pi}{3} + 2\pi m$$

Pg.63

3. $h(t) = \cot t$

The average rate of change of a function $y = f(x)$ over the interval $[a, b]$ is $\dfrac{f(b)-f(a)}{b-a}$.

(a) $\left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$

$$\frac{\cot\left(\frac{3\pi}{4}\right)-\cot\left(\frac{\pi}{4}\right)}{\frac{3\pi}{4}-\frac{\pi}{4}} = \frac{-1-1}{\frac{\pi}{2}} = \frac{-2}{\frac{\pi}{2}} = -\frac{4}{\pi}$$

(b) $\left[\dfrac{\pi}{6}, \dfrac{\pi}{2}\right]$

$$\frac{\cot\left(\frac{\pi}{2}\right)-\cot\left(\frac{\pi}{6}\right)}{\frac{\pi}{2}-\frac{\pi}{6}} = \frac{0-\sqrt{3}}{\frac{2\pi}{6}} = -\frac{\sqrt{3}}{\frac{\pi}{3}} = -\frac{3\sqrt{3}}{\pi}$$

4. $g(t) = 2 + \cos t$

The average rate of change of a function $y = f(x)$ over the interval $[a, b]$ is $\dfrac{f(b)-f(a)}{b-a}$.

(a) $[0, \pi]$

$$\frac{2+\cos\pi-(2+\cos 0)}{\pi-0} = \frac{2+(-1)-(2+1)}{\pi} = \frac{1-3}{\pi} = -\frac{2}{\pi}$$

(a) $[-\pi, \pi]$

$$\frac{2+\cos\pi-(2+\cos(-\pi))}{\pi-(-\pi)} = \frac{2+(-1)-(2+(-1))}{2\pi} = \frac{1-1}{2\pi} = 0$$

8. Example 3 is needed
9. Example 3 is needed
13. Example 3 is needed
14. Example 3 is needed

15. $\lim_{x\to 2}\frac{x+3}{x+6}=\frac{\lim_{x\to 2}(x+3)}{\lim_{x\to 2}(x+6)}=\frac{2+3}{2+6}=\frac{5}{8}$

16.

$$\lim_{s\to 2/3}3s(2s-1)=\lim_{s\to 2/3}3\cdot\lim_{s\to 2/3}s\cdot\lim_{s\to 2/3}(2s-1)$$

$$=3\cdot\frac{2}{3}\cdot\left(2\cdot\frac{2}{3}-1\right)=2\cdot\left(\frac{4}{3}-1\right)$$

$$=2\cdot\frac{1}{3}=\frac{2}{3}$$

21.

$$\lim_{h\to 0}\frac{3}{\sqrt{3h+1}+1}=\frac{\lim_{h\to 0}3}{\lim_{h\to 0}(\sqrt{3h+1}+1)}=\frac{3}{\lim_{h\to 0}\sqrt{3h+1}+\lim_{h\to 0}1}$$

$$=\frac{3}{\sqrt{\lim_{h\to 0}(3h)+\lim_{h\to 0}1}+1}=\frac{3}{\sqrt{0+1}+1}=\frac{3}{2}$$

22.

$$\lim_{h\to 0}\frac{\sqrt{5h+4}-2}{h}=\lim_{h\to 0}\frac{(\sqrt{5h+4}-2)(\sqrt{5h+4}+2)}{h(\sqrt{5h+4}+2)}$$

$$=\lim_{h\to 0}\frac{5h+4-4}{h(\sqrt{5h+4}+2)}=\lim_{h\to 0}\frac{5h}{h(\sqrt{5h+4}+2)}$$

$$=\lim_{h\to 0}\frac{5}{\sqrt{5h+4}+2}=\frac{5}{\sqrt{5\cdot 0+4}+2}$$

$$=\frac{5}{\sqrt{4}+2}=\frac{5}{4}$$

27.

$$\lim_{t\to 1}\frac{t^2+t-2}{t^2-1}=\lim_{t\to 1}\frac{t^2+2t-t-2}{t^2-1}=\lim_{t\to 1}\frac{t(t+2)-(t+2)}{(t-1)(t+1)}$$

$$=\lim_{t\to 1}\frac{(t+2)(t-1)}{(t-1)(t+1)}=\lim_{t\to 1}\frac{t+2}{t+1}$$

$$=\frac{1+2}{1+1}=\frac{3}{2}$$

28.

$$\lim_{t\to -1}\frac{t^2+3t+2}{t^2-t-2}=\lim_{t\to -1}\frac{t^2+t+2t+2}{t^2-2t+t-2}=\lim_{t\to -1}\frac{t(t+1)+2(t+1)}{t(t-2)+(t-2)}$$

$$=\lim_{t\to -1}\frac{(t+1)(t+2)}{(t-2)(t+1)}=\lim_{t\to -1}\frac{t+2}{t-2}=\frac{-1+2}{-1-2}$$

$$=-\frac{1}{3}$$

33.

$$\lim_{u\to 1}\frac{u^4-1}{u^3-1}=\lim_{u\to 1}\frac{(u^2-1)(u^2+1)}{(u-1)(u^2+u+1)}=\lim_{u\to 1}\frac{(u-1)(u+1)(u^2+1)}{(u-1)(u^2+u+1)}$$

$$=\lim_{u\to 1}\frac{(u+1)(u^2+1)}{u^2+u+1}=\frac{(1+1)(1^2+1)}{1^2+1+1}$$

$$=\frac{4}{3}$$

34.

$$\lim_{v\to 2}\frac{v^3-8}{v^4-16}=\lim_{v\to 2}\frac{(v-2)(v^2+2v+4)}{(v^2-4)(v^2+4)}=\lim_{v\to 2}\frac{(v-2)(v^2+2v+4)}{(v-2)(v+2)(v^2+4)}$$

$$=\lim_{v\to 2}\frac{v^2+2v+4}{(v+2)(v^2+4)}=\frac{2^2+2\cdot 2+4}{(2+2)(2^2+4)}$$

$$=\frac{12}{32}=\frac{3}{8}$$

45. $\lim_{x\to 0}\sec x=\lim_{x\to 0}\frac{1}{\cos x}=\frac{1}{\cos 0}=\frac{1}{1}=1$

41.

$$\lim_{x\to -3}\frac{2-\sqrt{x^2-5}}{x+3}=\lim_{x\to -3}\frac{(2-\sqrt{x^2-5})(2+\sqrt{x^2-5})}{(x+3)(2+\sqrt{x^2-5})}$$

$$=\lim_{x\to -3}\frac{4-(x^2-5)}{(x+3)(2+\sqrt{x^2-5})}=\lim_{x\to -3}\frac{4-x^2+5}{(x+3)(2+\sqrt{x^2-5})}$$

$$= \lim_{x\to -3} \frac{9-x^2}{(x+3)(2+\sqrt{x^2-5})} = \lim_{x\to -3} \frac{(3-x)(3+x)}{(x+3)(2+\sqrt{x^2-5})}$$
$$= \lim_{x\to -3} \frac{3-x}{2+\sqrt{x^2-5}} = \frac{3-(-3)}{2+\sqrt{(-3)^2-5}}$$
$$= \frac{6}{2+\sqrt{4}} = \frac{6}{4} = \frac{3}{2}$$

49.

$$\lim_{x\to -\pi} \sqrt{x+4}\cos(x+\pi) = \lim_{x\to -\pi} \sqrt{x+4} \cdot \lim_{x\to -\pi} \cos(x+\pi)$$
$$= \sqrt{-\pi+4}\cos(-\pi+\pi) = \sqrt{4-\pi}\cos 0$$
$$= \sqrt{4-\pi} \cdot 1$$
$$= \sqrt{4-\pi}$$

56. $\lim_{x\to -2} p(x) = 4$, $\lim_{x\to -2} r(x) = 0$, $\lim_{x\to -2} s(x) = -3$

(a) $\lim_{x\to -2}(p(x)+r(x)+s(x)) = \lim_{x\to -2} p(x) + \lim_{x\to -2} r(x) + \lim_{x\to -2} s(x) = 4+0+(-3) = 1$

(b) $\lim_{x\to -2}(p(x)\cdot r(x)\cdot s(x)) = \lim_{x\to -2} p(x) \cdot \lim_{x\to -2} r(x) \cdot \lim_{x\to -2} s(x) = 4\cdot 0\cdot(-3) = 0$

(c)

$$\lim_{x\to -2} \frac{(-4p(x)+5r(x))}{s(x)} = \frac{\lim_{x\to -2}(-4p(x)+5r(x))}{\lim_{x\to -2} s(x)} = \frac{\lim_{x\to -2}(-4p(x)) + \lim_{x\to -2}(5r(x))}{-3}$$
$$= \frac{-4\lim_{x\to -2} p(x) + 5\lim_{x\to -2} r(x)}{-3} = \frac{-4\cdot 4+5\cdot 0}{-3} = \frac{16}{3}$$

57. $f(x) = x^2$, $x = 1$

$$m(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-x^2}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h}$$
$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0}(2x+h) = 2x$$

At x = 1 we have

$m(1) = 2(1) = 2$

58. $f(x) = x^2$, $x = -2$

$$m(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

$$= \lim_{h\to 0}\frac{2xh+h^2}{h} = \lim_{h\to 0}(2x+h) = 2x$$

At x = –2 we have

$m(-2) = 2(-2) = -4$

61. $f(x) = \sqrt{x}$, x = 7

$$m(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

At x = 7 we have

$$m(7) = \frac{1}{2\sqrt{7}} = \frac{\sqrt{7}}{14}$$

62. $f(x) = \sqrt{3x+1}$, x = 0

$$m(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{3(x+h)+1}-\sqrt{3x+1}}{h}$$

$$= \lim_{h\to 0}\frac{(\sqrt{3(x+h)+1}-\sqrt{3x+1})(\sqrt{3(x+h)+1}+\sqrt{3x+1})}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})}$$

$$= \lim_{h\to 0}\frac{3(x+h)+1-(3x+1)}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})} = \lim_{h\to 0}\frac{3x+3h+1-3x-1}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})}$$

$$= \lim_{h\to 0}\frac{3h}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})} = \lim_{h\to 0}\frac{3}{\sqrt{3(x+h)+1}+\sqrt{3x+1}}$$

$$= \frac{3}{\sqrt{3x+1}+\sqrt{3x+1}} = \frac{3}{2\sqrt{3x+1}}$$

At x = 0 we have

$$m(0) = \frac{3}{2\sqrt{3\cdot 0+1}} = \frac{3}{2}$$

Pg.82

9. It follows from the graph that

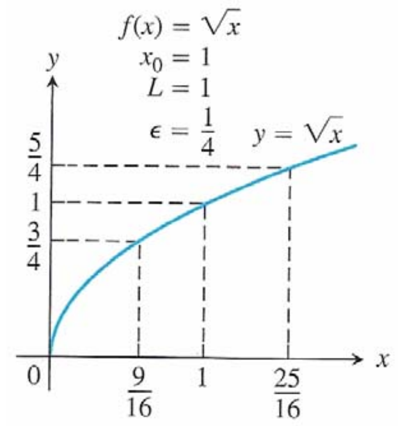
if $\frac{9}{16} < x < \frac{25}{16}$ then $\frac{3}{4} < \sqrt{x} < \frac{5}{4}$.

The interval $\left(\frac{9}{16}, \frac{25}{16}\right)$ is not symmetric about x = 1:

$\frac{25}{16} - 1 = \frac{9}{16}$

$1 - \frac{9}{16} = \frac{7}{16}$

We can choose δ to be the smaller of these numbers, that is $\delta = \frac{7}{16}$.



10. It follows from the graph that

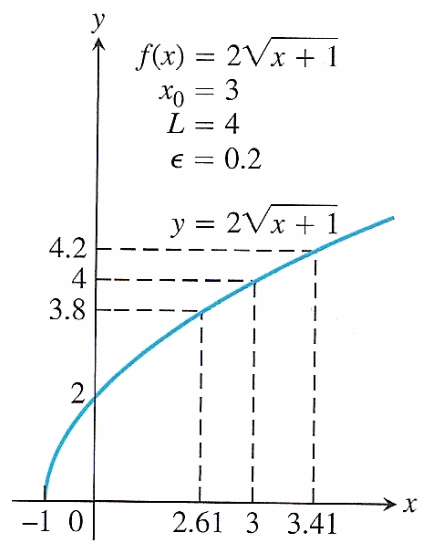
if 2.61 < x < 3.41 then 3.8 < $2\sqrt{x+1}$ < 4.2

The interval (2.61, 3.41) is not symmetric about x = 3:

3.41 – 3 = 0.41

3 – 2.61 = 0.39

We can choose δ to be the smaller of these numbers, that is δ = 0.39



20. $f(x)=\sqrt{x-7}$, L = 4, x_0 = 23, ε = 1

Step 1: Solve the inequality $|f(x)-L|<\varepsilon$ to find the interval about x_0 = 23 on which the inequality holds for all $x \neq x_0$:

$|\sqrt{x-7}-4|<1$

$-1<\sqrt{x-7}-4<1$

$3<\sqrt{x-7}<5$

$9<x-7<25$

$16<x<32$

Step 2: Find a value of δ > 0 that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval 16 < x < 32:

32 – 23 = 9

23 – 16 = 7

The distance from x_0 = 23 to the nearer endpoint of (16, 32) is 7. If we take δ = 7 or any smaller positive number, then

$0<|x-23|<7 \quad \Rightarrow \quad |\sqrt{x-7}-4|<1$

21. $f(x)=\frac{1}{x}$, $L=\frac{1}{4}$, x_0 = 4, ε = 0.05

Step 1: Solve the inequality $|f(x)-L|<\varepsilon$ to find the interval about x_0 = 4 on which the inequality holds for all $x \neq x_0$:

$\left|\frac{1}{x}-\frac{1}{4}\right|<0.05$

$-0.05<\frac{1}{x}-\frac{1}{4}<0.05$

$0.2<\frac{1}{x}<0.3$

$\frac{10}{3}<x<5$

Step 2: Find a value of δ > 0 that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval $\frac{10}{3}<x<5$:

5 – 4 = 1

$4-\frac{10}{3}=\frac{2}{3}$

The distance from $x_0 = 4$ to the nearer endpoint of $\left(\frac{10}{3}, 5\right)$ is $\frac{2}{3}$. If we take $\delta = \frac{2}{3}$ or any smaller positive number, then

$0 < |x-4| < \frac{2}{3} \quad \Rightarrow \quad \left|\frac{1}{x} - \frac{1}{4}\right| < 0.05$

25. $f(x) = x^2 - 5$, $L = 11$, $x_0 = 4$, $\varepsilon = 1$

Step 1: Solve the inequality $|f(x) - L| < \varepsilon$ to find the interval about $x_0 = 4$ on which the inequality holds for all $x \neq x_0$:

$|x^2 - 5 - 11| < 1$

$-1 < x^2 - 16 < 1$

$15 < x^2 < 17$

$\sqrt{15} < x < \sqrt{17}$

Step 2: Find a value of $\delta > 0$ that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval $\sqrt{15} < x < \sqrt{17}$:

$\sqrt{17} - 4 \approx 0.1231$

$4 - \sqrt{15} \approx 0.1270$

The distance from $x_0 = 4$ to the nearer endpoint of $\left(\sqrt{15}, \sqrt{17}\right)$ is $\sqrt{17} - 4$. If we take $\delta = \sqrt{17} - 4$ or any smaller positive number, then

$0 < |x-4| < \sqrt{17} - 4 \quad \Rightarrow \quad |x^2 - 5 - 11| = |x^2 - 16| < 1$

33. $f(x) = \frac{x^2 - 4}{x - 2}$, $x_0 = 2$, $\varepsilon = 0.05$

$$L = \lim_{x \to x_0} f(x) = \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \to 2}(x+2) = 2 + 2 = 4$$

Step 1: Solve the inequality $|f(x) - L| < \varepsilon$ to find the interval about $x_0 = 4$ on which the inequality holds for all $x \neq x_0$:

$\left|\frac{x^2 - 4}{x - 2} - 4\right| < 0.05$

$-0.05 < \frac{x^2 - 4}{x - 2} - 4 < 0.05$

$$3.95 < \frac{x^2-4}{x-2} < 4.05$$

$$3.95 < \frac{(x-2)(x+2)}{x-2} < 4.05$$

$$3.95 < x+2 < 4.05$$

$$1.95 < x < 2.05$$

Step 2: Find a value of δ > 0 that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval $1.95 < x < 2.05$:

$2.05 - 2 = 0.05$

$2 - 1.95 = 0.05$

The distance from $x_0 = 2$ to the nearer endpoint of (1.95, 2.05) is 0.05. If we take $\delta = 0.05$ or any smaller positive number, then

$$0 < |x-2| < 0.05 \qquad \Rightarrow \qquad \left|\frac{x^2-4}{x-2} - 4\right| < 0.05$$

34. $f(x) = \dfrac{x^2+6x+5}{x+5}$, $x_0 = -5$, $\varepsilon = 0.05$

$$L = \lim_{x\to x_0} f(x) = \lim_{x\to -5}\frac{x^2+6x+5}{x+5} = \lim_{x\to -5}\frac{x^2+x+5x+5}{x+5} = \lim_{x\to -5}\frac{x(x+1)+5(x+1)}{x+5}$$

$$= \lim_{x\to -5}\frac{(x+1)(x+5)}{x+5} = \lim_{x\to -5}(x+1) = -5+1 = -4$$

Step 1: Solve the inequality $|f(x) - L| < \varepsilon$ to find the interval about $x_0 = 4$ on which the inequality holds for all $x \neq x_0$:

$$\left|\frac{x^2+6x+5}{x+5} - (-4)\right| < 0.05$$

$$-0.05 < \frac{x^2+6x+5}{x+5} + 4 < 0.05$$

$$-4.05 < \frac{(x+5)(x+1)}{x+5} < -3.95$$

$$-4.05 < x+1 < -3.95$$

$$-5.05 < x < -4.95$$

Step 2: Find a value of δ > 0 that places the centered interval $x_0 - \delta < x < x_0 + \delta$ inside the interval $-5.05 < x < -4.95$:

$-5 - (-5.05) = 0.05$

$-4.95 - (-5) = 0.05$

The distance from $x_0 = -5$ to the nearer endpoint of (–5.05, –4.95) is 0.05. If we take δ = 0.05 or any smaller positive number, then

$$0 < |x+5| < 0.05 \qquad \Rightarrow \qquad \left|\frac{x^2+6x+5}{x+5}+4\right| < 0.05$$

43. $\lim_{x\to 1}\frac{1}{x} = 1$

Given ε > 0, we want to find δ > 0 such that

if $\quad 0 < |x-1| < \delta \qquad$ then $\qquad \left|\frac{1}{x}-1\right| < \varepsilon$

We have

$$\left|\frac{1}{x}-1\right| = \left|\frac{1-x}{x}\right| = \frac{|1-x|}{|x|} < \varepsilon$$

We find a positive constant C such that $\frac{1}{|x|} < C \quad \Rightarrow \quad \frac{|x-1|}{|x|} < C|x-1|$ and we can make

$C|x-1| < \varepsilon$ by taking $|x-1| < \frac{\varepsilon}{C} = \delta$

We restrict x to lie in the interval $|x-1| < \frac{1}{2}$, then

$$\frac{1}{2} < x < \frac{3}{2} \qquad \Rightarrow \qquad \frac{2}{3} < \frac{1}{x} < 2 \qquad \Rightarrow \qquad \frac{1}{|x|} < 2$$

So C = 2 is suitable and we should choose $\delta = \min\left\{\frac{1}{2}, \frac{\varepsilon}{2}\right\}$.

Show that δ works.

Given ε > 0, we let $\delta = \min\left\{\frac{1}{2}, \frac{\varepsilon}{2}\right\}$. If $0 < |x-1| < \delta$ then

$$|x-1| < \frac{1}{2} \qquad \Rightarrow \qquad \frac{1}{2} < x < \frac{3}{2} \qquad \Rightarrow \qquad \frac{1}{|x|} < 2$$

Also

$$\frac{|1-x|}{|x|} < \varepsilon \qquad \Rightarrow \qquad 2|1-x| < \varepsilon \qquad \Rightarrow \qquad |1-x| < \frac{\varepsilon}{2}$$

so

$$\left|\frac{1}{x}-1\right| = \frac{|1-x|}{|x|} < 2\cdot\frac{\varepsilon}{2} = \varepsilon$$

Therefore,

$$\lim_{x\to 1}\frac{1}{x} = 1$$

44. $\lim_{x\to\sqrt{3}} \frac{1}{x^2} = \frac{1}{3}$

Given ε > 0, we want to find δ > 0 such that

if $\quad 0 < |x - \sqrt{3}| < \delta \qquad$ then $\qquad \left|\frac{1}{x^2} - \frac{1}{3}\right| < \varepsilon$

We have

$$\left|\frac{1}{x^2} - \frac{1}{3}\right| = \left|\frac{3 - x^2}{3x^2}\right| = \left|\frac{(\sqrt{3} - x)(\sqrt{3} + x)}{3x^2}\right| = \frac{|\sqrt{3} - x)||\sqrt{3} + x|}{|3x^2|} < \varepsilon$$

Let's assume that δ ≤ 1. This is a valid assumption to make since, in general, once we find a δ that works, all smaller values of δ also work. Then $|x - \sqrt{3}| < \delta \le 1$ implies that

$$-1 < x - \sqrt{3} < 1$$

$$-1 + \sqrt{3} < x < 1 + \sqrt{3}$$

$$-1 + 2\sqrt{3} < x + \sqrt{3} < 1 + 2\sqrt{3}$$

Then

$$|\sqrt{3} - x)||\sqrt{3} + x| < |\sqrt{3} - x)|(1 + 2\sqrt{3}) < \varepsilon$$

$$|\sqrt{3} - x| < \frac{\varepsilon}{(1 + 2\sqrt{3})}$$

Now choose $\delta = \min\left\{1, \frac{\varepsilon}{(1 + 2\sqrt{3})}\right\}$. This guaranties that both assumptions made about δ in the course of this proof are taken into account simultaneously. Thus, if $0 < |x - \sqrt{3}| < \delta$, it follows that $\left|\frac{1}{x^2} - \frac{1}{3}\right| < \varepsilon$. This completes the proof.

Pg.90

7.

(a)

$$f(x) = \begin{cases} x^3, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

Find the x-intercept:

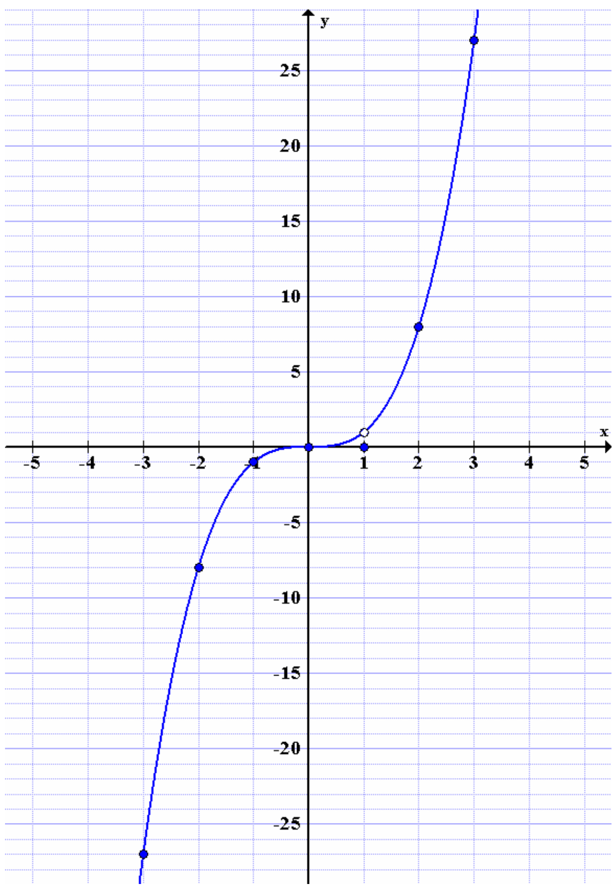
$x^3 = 0$

$x = 0$

The x-intercept is (0, 0).

Find few points:

x	–3	–2	–1	1	2	3
$y = x^3$	–27	–8	–1	1	8	27



(b)

$$\lim_{x \to 1^-} f(x) = 1$$

$$\lim_{x \to 1^+} f(x) = 1$$

(c) Since $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$ then $\lim_{x \to 1} f(x)$ exists:

$\lim_{x \to 1} f(x) = 1$

8. $f(x) = \begin{cases} 1 - x^2, & x \neq 1 \\ 2, & x = 1 \end{cases}$

(a) Find the x-intercepts:

$1 - x^2 = 0$

$(1 - x)(1 + x) = 0$

$1 - x = 0$ or $1 + x = 0$

$x = 1$ or $x = -1$

The x-intercept is (–1, 0) (the point (1, 0) is not on the graph of *f*).

Find the y-intercept:

$f(0) = 1 - 0^2 = 1$

The y-intercept is (0, 1).

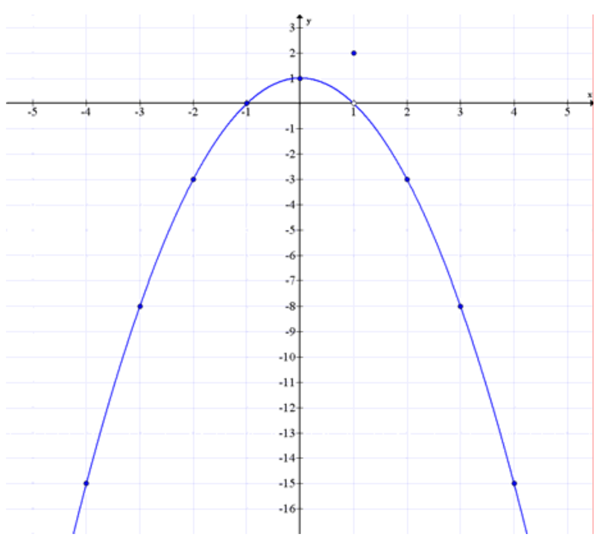
The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{0}{2 \cdot (-1)} = 0$$

The vertex is (0, 1).

Find few points:

x	–4	–3	–2	2	3	4
$y = x^3$	–15	–8	–3	–3	–8	–15



(b)

$$\lim_{x\to1^-} f(x)=0$$

$$\lim_{x\to1^+} f(x)=0$$

(c) Since $\lim_{x\to1^-} f(x)=\lim_{x\to1^+} f(x)$ then $\lim_{x\to1} f(x)$ exists:

$$\lim_{x\to1} f(x)=0$$

11. $\lim_{x\to-0.5^-}\sqrt{\frac{x+2}{x+1}}=\sqrt{\frac{-0.5+2}{-0.5+1}}=\sqrt{\frac{1.5}{0.5}}=\sqrt{3}$

12. $\lim_{x\to1^+}\sqrt{\frac{x-1}{x+2}}$

As $x\to1^+$, we have $(x-1)\to0^+$ and $(x+2)\to3^+$,so

$$\lim_{x\to1^+}\sqrt{\frac{x-1}{x+2}}=0$$

13.

$$\lim_{x\to-2^+}\left(\frac{x}{x+1}\right)\left(\frac{2x+5}{x^2+x}\right)=\lim_{x\to-2^+}\frac{x(2x+5)}{(x+1)(x^2+x)}=\lim_{x\to-2^+}\frac{x(2x+5)}{x(x+1)(x+1)}$$

$$=\lim_{x\to-2^+}\frac{2x+5}{(x+1)^2}=\frac{2(-2)+5}{(-2+1)^2}=\frac{1}{(-1)^2}=1$$

25.

$$\lim_{x\to0}\frac{\tan 2x}{x}=\lim_{x\to0}\frac{\frac{\sin 2x}{\cos 2x}}{x}=\lim_{x\to0}\left(\frac{\sin 2x}{x}\cdot\frac{1}{\cos 2x}\right)=2\lim_{x\to0}\left(\frac{\sin 2x}{2x}\cdot\frac{1}{\cos 2x}\right)$$

$$=2\lim_{x\to0}\frac{\sin 2x}{2x}\cdot\lim_{x\to0}\frac{1}{\cos 2x}=2\cdot1\cdot\frac{1}{\cos 0}=2\cdot\frac{1}{1}$$

$$=2$$

26.

$$\lim_{t\to0}\frac{2\,\mathrm{t}}{\tan t}=\lim_{t\to0}\frac{2\,\mathrm{t}}{\frac{\sin t}{\cos t}}=\lim_{t\to0}\frac{2\,\mathrm{t}\cos t}{\sin t}=\lim_{t\to0}\frac{2\cos t}{\frac{\sin t}{t}}$$

$$=\frac{2\lim_{t\to0}\cos t}{\lim_{t\to0}\frac{\sin t}{t}}=\frac{2\cos 0}{1}=2$$

39.

$$\lim_{x\to 0}\frac{\tan 3x}{\sin 8x}=\lim_{x\to 0}\left(\frac{\frac{\sin 3x}{\cos 3x}}{8x\cdot\frac{\sin 8x}{8x}}\right)=\lim_{x\to 0}\left(\frac{1}{8x}\frac{\sin 3x}{\frac{\sin 8x}{8x}}\cdot\frac{1}{\cos 3x}\right)=\lim_{x\to 0}\left(\frac{1}{8x}\frac{3x\cdot\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8}}\cdot\frac{1}{\cos 3x}\right)$$

$$=\lim_{x\to 0}\left(\frac{3x}{8x}\frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8}}\cdot\frac{1}{\cos 3x}\right)=\lim_{x\to 0}\left(\frac{3}{8}\frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8x}}\cdot\frac{1}{\cos 3x}\right)=\frac{3}{8}\lim_{x\to 0}\left(\frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8x}}\cdot\frac{1}{\cos 3x}\right)$$

$$=\frac{3}{8}\left(\lim_{x\to 0}\frac{\frac{\sin 3x}{3x}}{\frac{\sin 8x}{8x}}\cdot\lim_{x\to 0}\frac{1}{\cos 3x}\right)=\frac{3}{8}\left(\frac{\lim_{x\to 0}\frac{\sin 3x}{3x}}{\lim_{x\to 0}\frac{\sin 8x}{8x}}\cdot\frac{1}{\cos 0}\right)$$

$$=\frac{3}{8}\left(\frac{1}{1}\cdot\frac{1}{1}\right)=\frac{3}{8}$$

40.

$$\lim_{y\to 0}\frac{\sin 3y\cot 5y}{y\cot 4y}=\lim_{y\to 0}\left(\frac{\sin 3y}{y}\cdot\frac{\cot 5y}{\cot 4y}\right)=\lim_{y\to 0}\left(3\cdot\frac{\sin 3y}{3y}\cdot\frac{\frac{\cos 5y}{\sin 5y}}{\frac{\cos 4y}{\sin 4y}}\right)$$

$$=3\lim_{y\to 0}\left(\frac{\sin 3y}{3y}\cdot\frac{\cos 5y}{\sin 5y}\cdot\frac{\sin 4y}{\cos 4y}\right)=3\lim_{y\to 0}\left(\frac{\sin 3y}{3y}\cdot\frac{\sin 4y}{\sin 5y}\cdot\frac{\cos 5y}{\cos 4y}\right)$$

$$=3\left(\lim_{y\to 0}\frac{\sin 3y}{3y}\cdot\lim_{y\to 0}\frac{\sin 4y}{\sin 5y}\cdot\lim_{y\to 0}\frac{\cos 5y}{\cos 4y}\right)=3\left(1\cdot\lim_{y\to 0}\frac{4y\cdot\frac{\sin 4y}{4y}}{5y\cdot\frac{\sin 5y}{5y}}\cdot\frac{\cos 0}{\cos 0}\right)$$

$$=3\left(\lim_{y\to 0}\left(\frac{4y}{5y}\frac{\frac{\sin 4y}{4y}}{\frac{\sin 5y}{5y}}\right)\cdot\frac{1}{1}\right)=3\lim_{y\to 0}\left(\frac{4}{5}\frac{\frac{\sin 4y}{4y}}{\frac{\sin 5y}{5y}}\right)=\frac{12}{5}\lim_{y\to 0}\left(\frac{\frac{\sin 4y}{4y}}{\frac{\sin 5y}{5y}}\right)$$

$$=\frac{12}{5}\cdot\frac{\lim_{y\to 0}\frac{\sin 4y}{4y}}{\lim_{y\to 0}\frac{\sin 5y}{5y}}=\frac{12}{5}\cdot\frac{1}{1}=\frac{12}{5}$$

41.

$$\lim_{\theta\to 0}\frac{\tan\theta}{\theta^2\cot 3\theta}=\lim_{\theta\to 0}\frac{\frac{\sin\theta}{\cos\theta}}{\theta^2\frac{\cos 3\theta}{\sin 3\theta}}=\lim_{\theta\to 0}\frac{\sin\theta\sin 3\theta}{\theta^2\cos\theta\cos 3\theta}=\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\cdot\frac{\sin 3\theta}{\theta}\cdot\frac{1}{\cos\theta\cos 3\theta}\right)$$

$$=\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\lim_{\theta\to 0}\frac{\sin 3\theta}{\theta}\cdot\lim_{\theta\to 0}\frac{1}{\cos\theta\cos 3\theta}=1\cdot 3\lim_{\theta\to 0}\frac{\sin 3\theta}{3\theta}\cdot\frac{1}{\cos 0\cos 0}$$

$$=3\cdot 1\cdot\frac{1}{1\cdot 1}=3$$

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13. $y=\frac{1}{x-2}-3x$

The first term is a rational function so it is continuous everywhere except x = 2 (it is not defined at x = 2), the second term is a polynomial function so it is continuous everywhere. Therefore, *y* is continuous on the intervals $(-\infty, 2)\cup(2, \infty)$.

14. $y=\frac{1}{(x+2)^2}+4$

The first term is a rational function so it is continuous everywhere except x = −2 (it is not defined at x = −2), the second term is a constant function so it is continuous everywhere. Therefore, *y* is continuous on the intervals $(-\infty,-2)\cup(-2, \infty)$.

15. $y=\frac{x+1}{x^2-4x+3}$

Find the domain:

$$x^2-4x+3=0$$
$$x^2-3x-x+3=0$$
$$x(x-3)-(x-3)=0$$
$$(x-3)(x-1)=0$$
$$x=3 \quad or \quad x=1$$

The domain is the intervals $(-\infty, 1)\cup(1, 3)\cup(3, \infty)$.
y is a rational function so it is continuous on its domain or on the intervals $(-\infty, 1)\cup(1, 3)\cup(3, \infty)$.

35.

$$\lim_{t\to 0}\cos\left(\frac{\pi}{\sqrt{19-3\sec 2t}}\right)=\cos\left(\frac{\pi}{\sqrt{19-3\sec 0}}\right)=\cos\left(\frac{\pi}{\sqrt{19-3\cdot 1}}\right)$$

$$=\cos\left(\frac{\pi}{\sqrt{16}}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{\sqrt{19-3\sec 0}}\right)=\cos\left(\frac{\pi}{\sqrt{19-3\cdot 1}}\right)=\cos\left(\frac{\pi}{\sqrt{16}}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

Since $\lim_{t\to 0} f(t)=f(0)$ then the function is continuous at t = 0.

36. $\lim_{x\to\frac{\pi}{6}}\sqrt{\csc^2 x+5\sqrt{3}\tan x}=\sqrt{2^2+5\sqrt{3}\cdot\frac{\sqrt{3}}{3}}=\sqrt{4+5}=3$

$$\sqrt{\csc^2 x+5\sqrt{3}\tan x}=\sqrt{2^2+5\sqrt{3}\cdot\frac{\sqrt{3}}{3}}=\sqrt{4+5}=3$$

Since $\lim_{x\to\frac{\pi}{6}} f(x)=f\left(\frac{\pi}{6}\right)$ then the function is continuous at $x=\frac{\pi}{6}$.

37. $\lim_{x\to 0^+}\sin\left(\frac{\pi}{2}e^{\sqrt{x}}\right)=\sin\left(\frac{\pi}{2}e^{\sqrt{0}}\right)=\sin\left(\frac{\pi}{2}\cdot 1\right)=1$

$$\sin\left(\frac{\pi}{2}e^{\sqrt{0}}\right)=\sin\left(\frac{\pi}{2}\cdot 1\right)=1$$

Since $\lim_{x\to 0^+} f(x)=f(0)$ then the function is continuous from the right at x = 0.

39. $g(x)=\frac{x^2-9}{x-3}$

Find one-sided limits at x = 3:

$$\lim_{x\to 3^-}\frac{x^2-9}{x-3}=\lim_{x\to 3^-}\frac{(x-3)(x+3)}{x-3}=\lim_{x\to 3^-}(x+3)=3+3=6$$

$$\lim_{x\to 3^+}\frac{x^2-9}{x-3}=\lim_{x\to 3^+}\frac{(x-3)(x+3)}{x-3}=\lim_{x\to 3^+}(x+3)=3+3=6$$

Thus,

$$\lim_{x\to 3}\frac{x^2-9}{x-3}=6$$

To be continuous at x = 3 we must define g(3) = 6.

40. $h(t)=\dfrac{t^2+3t-10}{t-2}$

Find one-sided limits at t = 2:

$$\lim_{t\to 2^-}\frac{t^2+3t-10}{t-2}=\lim_{t\to 2^-}\frac{t^2+5t-2t-10}{t-2}=\lim_{t\to 2^-}\frac{t(t+5)-2(t+5)}{t-2}$$

$$=\lim_{t\to 2^-}\frac{(t+5)(t-2)}{t-2}=\lim_{t\to 2^-}(t+5)=2+5=7$$

$$\lim_{t\to 2^+}\frac{t^2+3t-10}{t-2}=\lim_{t\to 2^+}\frac{t^2+5t-2t-10}{t-2}=\lim_{t\to 2^+}\frac{t(t+5)-2(t+5)}{t-2}$$

$$=\lim_{t\to 2^+}\frac{(t+5)(t-2)}{t-2}=\lim_{t\to 2^+}(t+5)=2+5=7$$

Thus,

$$\lim_{t\to 2}\frac{t^2+3t-10}{t-2}=7$$

To be continuous at t = 2 we must define h(2) = 7.

41. $f(s)=\dfrac{s^3-1}{s^2-1}$

Find one-sided limits at s = 1:

$$\lim_{s\to 1^-}\frac{s^3-1}{s^2-1}=\lim_{s\to 1^-}\frac{(s-1)(s^2+s+1)}{(s-1)(s+1)}=\lim_{s\to 1^-}\frac{s^2+s+1}{s+1}=\frac{1^2+1+1}{1+1}=\frac{3}{2}$$

$$\lim_{s\to 1^+}\frac{s^3-1}{s^2-1}=\lim_{s\to 1^+}\frac{(s-1)(s^2+s+1)}{(s-1)(s+1)}=\lim_{s\to 1^+}\frac{s^2+s+1}{s+1}=\frac{1^2+1+1}{1+1}=\frac{3}{2}$$

Thus,

$$\lim_{s\to 1}\frac{s^3-1}{s^2-1}=\frac{3}{2}$$

To be continuous at s = 1 we must define $f(1)=\dfrac{3}{2}$.

55. $x^3 - 15x + 1 = 0$

Consider the function $f(x) = x^3 - 15x + 1$.

$f(-4) = (-4)^3 - 15(-4) + 1 = -3$

$f(-3) = (-3)^3 - 15(-3) + 1 = 19$

$f(-2) = (-2)^3 - 15(-2) + 1 = 23$

$f(-1) = (-1)^3 - 15(-1) + 1 = 15$

$f(0) = 0^3 - 15(0) + 1 = 1$

$f(1) = 1^3 - 15(1) + 1 = -13$

$f(2) = 2^3 - 15(2) + 1 = -21$

f(3) = 3^3 – 15(3) + 1 = –17
f(4) = 4^3 – 15(4) + 1 = 5

f is a polynomial function, so it is continuous everywhere. *f* changes its sign on the intervals (–4, –3), (0, 1) and (3, 4). By the Intermediate Value Theorem *f* must have three zeros on these intervals.

56. $F(x) = (x-a)^2(x-b)^2 + x$

$F(a) = (a-a)^2(a-b)^2 + a = 0 + a = a$
$F(b) = (b-a)^2(b-b)^2 + b = 0 + b = b$

The number $\frac{a+b}{2}$ lies between *a* and *b*, so by the Intermediate Value Theorem there exists a number *c* between *a* and *b* such that $f(c) = \frac{a+b}{2}$.

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16. $f(x) = \frac{3x+7}{x^2-2}$

(a) $\lim_{x\to\infty} \frac{3x+7}{x^2-2} = \lim_{x\to\infty} \frac{\frac{3x}{x^2}+\frac{7}{x^2}}{\frac{x^2}{x^2}-\frac{2}{x^2}} = \lim_{x\to\infty} \frac{\frac{3}{x}+\frac{7}{x^2}}{1-\frac{2}{x^2}} = \frac{0+0}{1-0} = 0$

(b) $\lim_{x\to-\infty} \frac{3x+7}{x^2-2} = \lim_{x\to-\infty} \frac{\frac{3x}{x^2}+\frac{7}{x^2}}{\frac{x^2}{x^2}-\frac{2}{x^2}} = \lim_{x\to\infty} \frac{\frac{3}{x}+\frac{7}{x^2}}{1-\frac{2}{x^2}} = \frac{0+0}{1-0} = 0$

17. $h(x) = \frac{7x^3}{x^3-3x^2+6x}$

(a) $\lim_{x\to\infty} \frac{7x^3}{x^3-3x^2+6x} = \lim_{x\to\infty} \frac{\frac{7x^3}{x^3}}{\frac{x^3}{x^3}-\frac{3x^2}{x^3}+\frac{6x}{x^3}} = \lim_{x\to\infty} \frac{7}{1-\frac{3}{x}+\frac{6}{x^2}} = \frac{7}{1-0+0} = 7$

(b) $\lim_{x\to-\infty}\frac{7x^3}{x^3-3x^2+6x}=\lim_{x\to-\infty}\frac{\frac{7x^3}{x^3}}{\frac{x^3}{x^3}-\frac{3x^2}{x^3}+\frac{6x}{x^3}}=\lim_{x\to-\infty}\frac{7}{1-\frac{3}{x}+\frac{6}{x^2}}=\frac{7}{1-0+0}=7$

19. $g(x)=\frac{10x^5+x^4+31}{x^6}$

(a) $\lim_{x\to\infty}\frac{10x^5+x^4+31}{x^6}=\lim_{x\to\infty}\frac{\frac{10x^5}{x^6}+\frac{x^4}{x^6}+\frac{31}{x^6}}{\frac{x^6}{x^6}}=\lim_{x\to\infty}\frac{\frac{10}{x}+\frac{1}{x^2}+\frac{31}{x^6}}{1}=\frac{0+0+0}{1}=0$

(b) $\lim_{x\to-\infty}\frac{10x^5+x^4+31}{x^6}=\lim_{x\to-\infty}\frac{\frac{10x^5}{x^6}+\frac{x^4}{x^6}+\frac{31}{x^6}}{\frac{x^6}{x^6}}=\lim_{x\to-\infty}\frac{\frac{10}{x}+\frac{1}{x^2}+\frac{31}{x^6}}{1}=\frac{0+0+0}{1}=0$

20. $h(x)=\frac{9x^4+x}{2x^4+5x^2-x+6}$

(a)

$$\lim_{x\to\infty}\frac{9x^4+x}{2x^4+5x^2-x+6}=\lim_{x\to\infty}\frac{\frac{9x^4}{x^4}+\frac{x}{x^4}}{\frac{2x^4}{x^4}+\frac{5x^2}{x^4}-\frac{x}{x^4}+\frac{6}{x^4}}=\lim_{x\to\infty}\frac{9+\frac{1}{x^3}}{2+\frac{5}{x^2}-\frac{1}{x^3}+\frac{6}{x^4}}$$

$$=\frac{9+0}{2+0-0+0}=\frac{9}{2}$$

(b)

$$\lim_{x\to-\infty}\frac{9x^4+x}{2x^4+5x^2-x+6}=\lim_{x\to-\infty}\frac{\frac{9x^4}{x^4}+\frac{x}{x^4}}{\frac{2x^4}{x^4}+\frac{5x^2}{x^4}-\frac{x}{x^4}+\frac{6}{x^4}}=\lim_{x\to-\infty}\frac{9+\frac{1}{x^3}}{2+\frac{5}{x^2}-\frac{1}{x^3}+\frac{6}{x^4}}$$

$$=\frac{9+0}{2+0-0+0}=\frac{9}{2}$$

22. $h(x)=\dfrac{-x^4}{x^4-7x^3+7x^2+9}$

(a) $\lim\limits_{x\to\infty}\dfrac{-x^4}{x^4-7x^3+7x^2+9}=\lim\limits_{x\to\infty}\dfrac{-\dfrac{x^4}{x^4}}{\dfrac{x^4}{x^4}-\dfrac{7x^3}{x^4}+\dfrac{7x^2}{x^4}+\dfrac{9}{x^4}}=\lim\limits_{x\to\infty}\dfrac{-1}{1-\dfrac{7}{x}+\dfrac{7}{x^2}+\dfrac{9}{x^4}}=\dfrac{-1}{1}=-1$

(b) $\lim\limits_{x\to-\infty}\dfrac{-x^4}{x^4-7x^3+7x^2+9}=\lim\limits_{x\to-\infty}\dfrac{-\dfrac{x^4}{x^4}}{\dfrac{x^4}{x^4}-\dfrac{7x^3}{x^4}+\dfrac{7x^2}{x^4}+\dfrac{9}{x^4}}=\lim\limits_{x\to-\infty}\dfrac{-1}{1-\dfrac{7}{x}+\dfrac{7}{x^2}+\dfrac{9}{x^4}}=\dfrac{-1}{1}=-1$

24.

$$\lim_{x\to-\infty}\left(\frac{x^2+x-1}{8x^2-3}\right)^{\frac{1}{3}}=\lim_{x\to-\infty}\left(\frac{\dfrac{x^2}{x^2}+\dfrac{x}{x^2}-\dfrac{1}{x^2}}{\dfrac{8x^2}{x^2}-\dfrac{3}{x^2}}\right)^{\frac{1}{3}}=\lim_{x\to-\infty}\left(\frac{1+\dfrac{1}{x}-\dfrac{1}{x^2}}{8-\dfrac{3}{x^2}}\right)^{\frac{1}{3}}$$

$$=\left(\frac{1+0-0}{8-0}\right)^{\frac{1}{3}}=\left(\frac{1}{8}\right)^{\frac{1}{3}}=\frac{1}{2}$$

29. $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}}=\lim\limits_{x\to-\infty}\dfrac{x^{\frac{1}{3}}-x^{\frac{1}{5}}}{x^{\frac{1}{3}}+x^{\frac{1}{5}}}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}-\dfrac{x^{\frac{1}{5}}}{x^{\frac{1}{3}}}}{\dfrac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}+\dfrac{x^{\frac{1}{5}}}{x^{\frac{1}{3}}}}=\lim\limits_{x\to-\infty}\dfrac{1-\dfrac{1}{x^{\frac{1}{3}-\frac{1}{5}}}}{1+\dfrac{1}{x^{\frac{1}{3}-\frac{1}{5}}}}=\lim\limits_{x\to-\infty}\dfrac{1-\dfrac{1}{x^{\frac{2}{15}}}}{1+\dfrac{1}{x^{\frac{2}{15}}}}=\dfrac{1-0}{1+0}=1$

30. $\lim\limits_{x\to\infty}\dfrac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{x^{-1}}{x^{-1}}-\dfrac{x^{-4}}{x^{-1}}}{\dfrac{x^{-2}}{x^{-1}}+\dfrac{x^{-3}}{x^{-1}}}=\lim\limits_{x\to-\infty}\dfrac{1-\dfrac{1}{x^{-1+4}}}{\dfrac{1}{x^{-1+2}}+\dfrac{1}{x^{-1+3}}}=\lim\limits_{x\to-\infty}\dfrac{1-\dfrac{1}{x^3}}{\dfrac{1}{x}+\dfrac{1}{x^2}}=\infty$

43. $\lim\limits_{x\to7}\dfrac{4}{(x-7)^2}=\infty$

because $(x-7)^2\to0$ as $x\to7$.

44. $\lim\limits_{x\to0}\dfrac{-1}{x^2(x+1)}=-\infty$

because $x^2\to0$ and $x+1\to1$ as $x\to0$.

55. (a) $\lim_{x\to 0^+}\left(\frac{x^2}{2}-\frac{1}{x}\right)=\lim_{x\to 0^+}\frac{x^2}{2}-\lim_{x\to 0^+}\frac{1}{x}=0-\lim_{x\to 0^+}\frac{1}{x}=-\infty$

(b) $\lim_{x\to 0^-}\left(\frac{x^2}{2}-\frac{1}{x}\right)=\lim_{x\to 0^-}\frac{x^2}{2}-\lim_{x\to 0^-}\frac{1}{x}=0-\lim_{x\to 0^-}\frac{1}{x}=\infty$

(c) $\lim_{x\to\sqrt[3]{2}}\left(\frac{x^2}{2}-\frac{1}{x}\right)=\frac{(\sqrt[3]{2})^2}{2}-\frac{1}{\sqrt[3]{2}}=\frac{\sqrt[3]{4}}{2}-\frac{1}{\sqrt[3]{2}}=\frac{\sqrt[3]{8}-2}{2\sqrt[3]{2}}=\frac{2-2}{2\sqrt[3]{2}}=0$

(d) $\lim_{x\to -1}\left(\frac{x^2}{2}-\frac{1}{x}\right)=\frac{(-1)^2}{2}-\frac{1}{-1}=\frac{1}{2}+1=\frac{3}{2}$

56.

(a) $\lim_{x\to -2^+}\frac{x^2-1}{2x+4}=\lim_{x\to -2^+}\frac{x^2-1}{2(x+2)}=\infty$

because $x^2-1\to 3$ and $\frac{1}{x+2}\to\infty$ as $x\to -2^+$.

(b) $\lim_{x\to -2^-}\frac{x^2-1}{2x+4}=\lim_{x\to -2^-}\frac{x^2-1}{2(x+2)}=-\infty$

because $x^2-1\to 3$ and $\frac{1}{x+2}\to -\infty$ as $x\to -2^-$.

(c) $\lim_{x\to 1^+}\frac{x^2-1}{2x+4}=\lim_{x\to 1^+}\frac{(x-1)(x+1)}{2x+4}=0$

because $x-1\to 0$, $x+1\to 2$ and $2x+4\to 6$ as $x\to 1^+$.

(d) $\lim_{x\to 0^-}\frac{x^2-1}{2x+4}=\frac{0^2-1}{2\cdot 0+4}=-\frac{1}{4}$

80.

$$\lim_{x\to\infty}\left(\sqrt{x+9}-\sqrt{x+4}\right)=\lim_{x\to\infty}\frac{(\sqrt{x+9}-\sqrt{x+4})(\sqrt{x+9}+\sqrt{x+4})}{\sqrt{x+9}+\sqrt{x+4}}$$

$$=\lim_{x\to\infty}\frac{x+9-(x+4)}{\sqrt{x+9}+\sqrt{x+4}}=\lim_{x\to\infty}\frac{x+9-x-4}{\sqrt{x+9}+\sqrt{x+4}}$$

$$=\lim_{x\to\infty}\frac{5}{\sqrt{x+9}+\sqrt{x+4}}=0$$

because $\sqrt{x+9}+\sqrt{x+4}\to\infty$ as $x\to\infty$.

81.

$$\lim_{x\to\infty}\left(\sqrt{x^2+25}-\sqrt{x^2-1}\right)=\lim_{x\to\infty}\frac{(\sqrt{x^2+25}-\sqrt{x^2-1})(\sqrt{x^2+25}+\sqrt{x^2-1})}{\sqrt{x^2+25}+\sqrt{x^2-1}}$$

$$=\lim_{x\to\infty}\frac{x^2+25-(x^2-1)}{\sqrt{x^2+25}+\sqrt{x^2-1}}=\lim_{x\to\infty}\frac{x^2+25-x^2+1}{\sqrt{x^2+25}+\sqrt{x^2-1}}$$

$$=\lim_{x\to\infty}\frac{26}{\sqrt{x^2+25}+\sqrt{x^2-1}}=0$$

because $\sqrt{x^2+25}+\sqrt{x^2-1}\to\infty$ as $x\to\infty$.

101. $y=\dfrac{x^2-4}{x-1}$

The domain is the intervals $(-\infty, 1)\cup(1, \infty)$.

$$y(-x)=\frac{(-x)^2-4}{-x-1}=-\frac{x^2-4}{x+1}$$

The function is neither odd nor even.

Find the x-intercepts:

$$\frac{x^2-4}{x-1}=0$$

$$x^2-4=0$$

$$(x-2)(x+2)=0$$

$$x=2 \quad or \quad x=-2$$

The x-intercepts are (–2, 0) and (2, 0).

Find the y-intercept:

$$y(0)=\frac{0^2-4}{0-1}=4$$

The y-intercept is (0, 4).

The denominator is zero at x = –1, so the line x = –1 is a vertical asymptote.

To find the equation of a slant asymptote we use the long division:

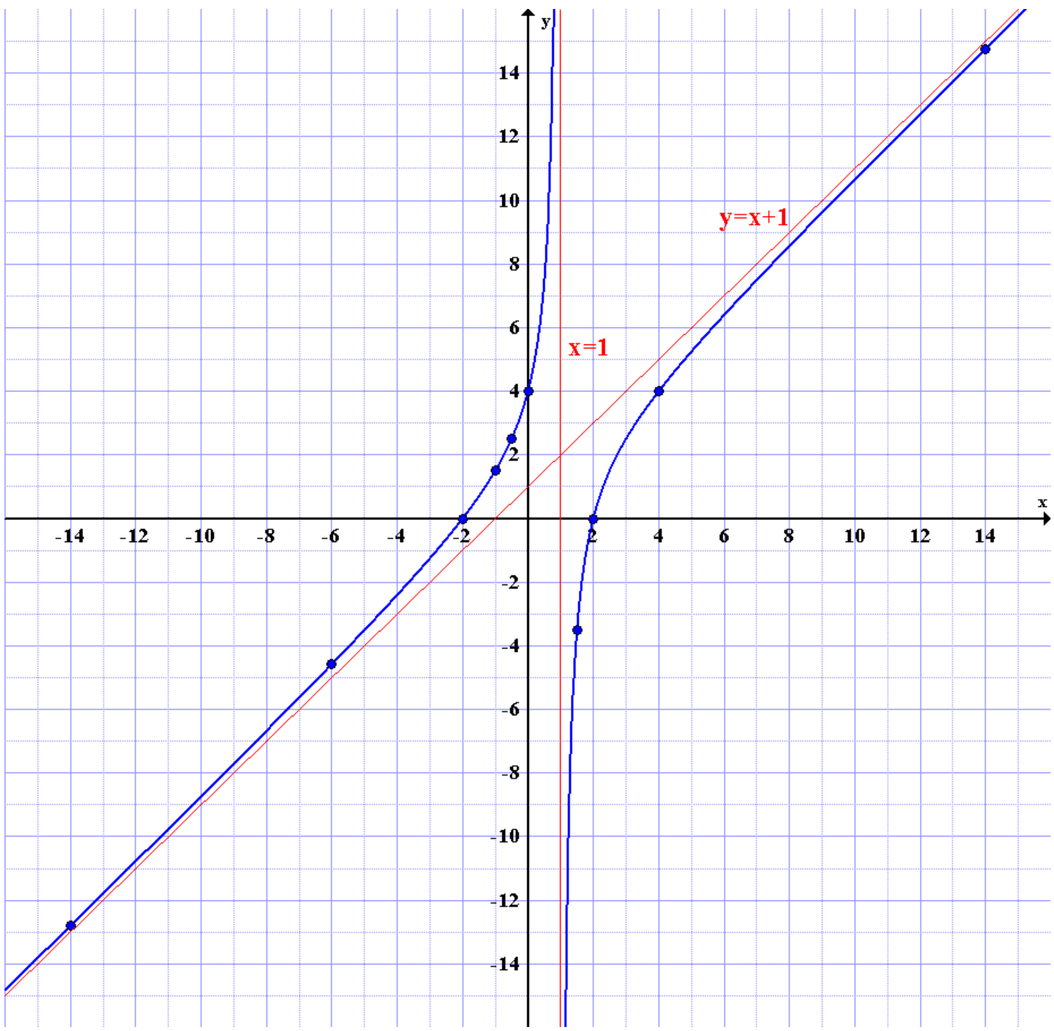
$$\begin{array}{r|l} & x+1 \\ \hline x-1 & x^2-4 \\ & \underline{x^2-x} \\ & \quad x-4 \\ & \quad \underline{x-1} \\ & \quad\ -3 \end{array}$$

Thus, $\dfrac{x^2-4}{x-1} = x+1-\dfrac{3}{x-1}$

The slant asymptote is y = x + 1

Find few points:

x	–14	–6	–1	$-\frac{1}{2}$	$\frac{3}{2}$	4	14
$y=\dfrac{x^2-4}{x-1}$	–12.8	–4.57	1.5	2.5	–3.5	4	14.77



102. $y = \dfrac{x^2 - 1}{2x+4}$

The domain is the intervals $(-\infty, -2) \cup (-2, \infty)$.

$$y(-x) = \frac{(-x)^2 - 1}{2(-x)+4} = \frac{x^2 - 1}{-2x+4}$$

The function is neither odd nor even.

Find the x-intercepts:

$$\frac{x^2-1}{2x+4} = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$x = 1 \quad or \quad x = -1$$

The x-intercepts are (–1, 0) and (1, 0).

Find the y-intercept:

$$y(0) = \frac{0^2 - 1}{2(0)+4} = -\frac{1}{4}$$

The y-intercept is $\left(0, -\dfrac{1}{4}\right)$.

The denominator is zero at x = –2, so the line x = –2 is a vertical asymptote.

To find the equation of a slant asymptote we use the long division:

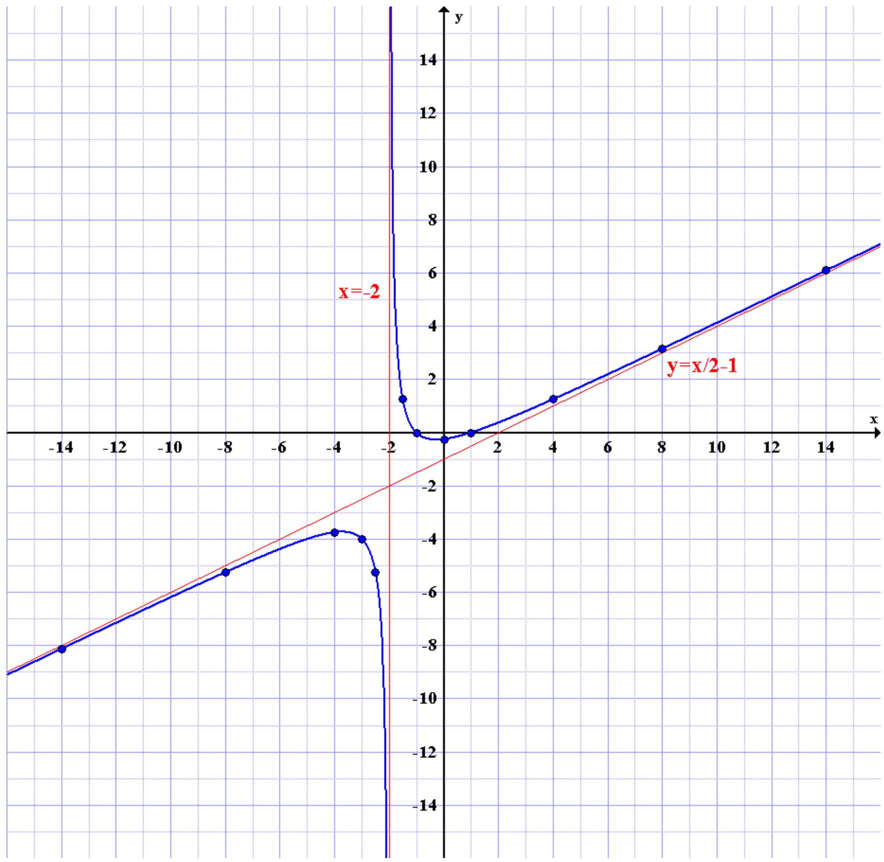
$$\begin{array}{r} \frac{1}{2}x - 1 \\ 2x+4 \overline{)\, x^2 - 1} \\ \underline{x^2 + 2x} \\ -2x - 1 \\ \underline{-2x - 4} \\ 3 \end{array}$$

Thus, $\dfrac{x^2 - 1}{2x+4} = \dfrac{1}{2}x - 1 + \dfrac{3}{2x+4}$

The slant asymptote is $y = \dfrac{1}{2}x - 1$.

Find few points:

x	–14	–8	–4	–3	$-\frac{5}{2}$	$-\frac{3}{2}$	4	8	14
$y = \frac{x^2-1}{2x+4}$	–8.13	–5.25	–3.75	–4	–5.25	1.25	1.25	3.15	6.09


y
x
x=-2
y=x/2-1
14
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-14