

Trigonometry Proofs (Show work)

1. $\sin 2\theta \cot \theta = 2 - 2\sin^2 \theta$

We start with the left-hand side:

$$\begin{aligned}\sin(2\theta) \cot \theta &= 2 \sin \theta \cos \theta \frac{\cos \theta}{\sin \theta} && \boxed{\sin(2\theta) = 2 \sin \theta \cos \theta} \quad , \quad \boxed{\cot \theta = \frac{\cos \theta}{\sin \theta}} \\ &= 2 \cos^2 \theta \\ &= 2(1 - \sin^2 \theta) && \boxed{\sin^2 \theta + \cos^2 \theta = 1} \Rightarrow \boxed{\cos^2 \theta = 1 - \sin^2 \theta} \\ &= 2 - 2\sin^2 \theta\end{aligned}$$

Having arrived at the right side, the identity is established.

Sketching trigonometric functions (Show work)

1. $f(x) = 4\cos(2x + \pi)$

Comparing $f(x) = 4\cos(2x + \pi)$ to $y = A \cos(\omega t - \phi)$, we find that the amplitude is $A = 4$
 $\omega = 2$ and $\phi = -\pi$. The period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$$

The graph of $f(x) = 4\cos(2x + \pi)$ will lie between -4 and 4 on the y -axis. One cycle will begin at $x = 0$ and end at $x = \pi$. We divide the interval $[0, \pi]$ into four subintervals, each of length $\frac{\pi}{4}$: $\left[0, \frac{\pi}{4}\right]$, $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$, $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$ and $\left[\frac{3\pi}{4}, \pi\right]$.

Calculate function at the endpoints of these intervals:

$$f(0) = 4 \cos(2 \cdot 0 + \pi) = 4 \cos \pi = 4(-1) = -4$$

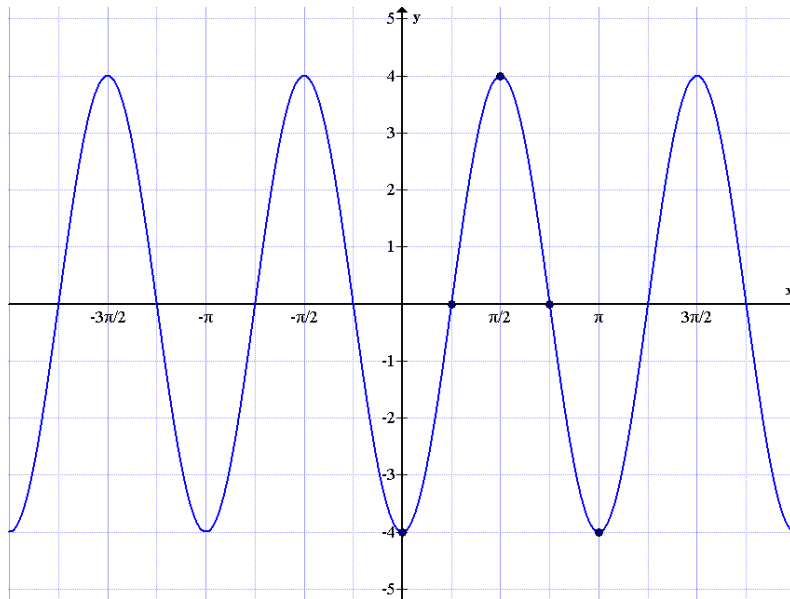
$$f\left(\frac{\pi}{4}\right) = 4 \cos\left(2 \cdot \frac{\pi}{4} + \pi\right) = 4 \cos\left(\frac{\pi}{2} + \pi\right) = 4 \cos\left(\frac{3\pi}{2}\right) = 4 \cdot 0 = 0$$

$$f\left(\frac{\pi}{2}\right) = 4 \cos\left(2 \cdot \frac{\pi}{2} + \pi\right) = 4 \cos(\pi + \pi) = 4 \cos(2\pi) = 4 \cdot 1 = 4$$

$$f\left(\frac{3\pi}{4}\right) = 4 \cos\left(2 \cdot \frac{3\pi}{4} + \pi\right) = 4 \cos\left(\frac{3\pi}{2} + \pi\right) = 4 \cos\left(\frac{5\pi}{2}\right) = 4 \cdot 0 = 0$$

$$f(\pi) = 4 \cos(2\pi + \pi) = 4 \cos(3\pi) = 4 \cdot (-1) = -4$$

We plot five points $(0, -4)$, $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, 4\right)$, $\left(\frac{3\pi}{4}, 0\right)$, $(\pi, -4)$ and extend the graph in either direction.



Solving Trigonometric Equations

Find all x , such $0 \leq x \leq 2\pi$

(Show work)

1. $\sin(2x) = \sin(x)$

$$\sin(2x) - \sin(x) = 0$$

$$2 \sin(x) \cos(x) - \sin(x) = 0$$

$$\boxed{\sin(2x) = 2 \sin(x) \cos(x)}$$

$$\sin(x)(2 \cos(x) - 1) = 0$$

$$\sin(x) = 0 \quad \text{or} \quad 2 \cos(x) - 1 = 0$$

$$\sin(x) = 0 \quad \text{or} \quad 2 \cos(x) = 1$$

$$\sin(x) = 0 \quad \text{or} \quad \cos(x) = \frac{1}{2}$$

$$x = \pi k, k \in \mathbb{Z} \quad \text{or} \quad x = \frac{\pi}{3} + 2\pi k, k \in \mathbb{Z} \quad \text{or} \quad x = \frac{5\pi}{3} + 2\pi k, k \in \mathbb{Z}$$

Find x in the interval $0 \leq x \leq 2\pi$:

$k = 0$:

$$x = \pi \cdot 0 = 0$$

$$x = \frac{\pi}{3} + 2\pi \cdot 0 = \frac{\pi}{3}$$

$$x = \frac{5\pi}{3} + 2\pi \cdot 0 = \frac{5\pi}{3}$$

$k = 1$:

$$x = \pi \cdot 1 = \pi$$

$$x = \frac{\pi}{3} + 2\pi \cdot 1 = \frac{7\pi}{3} > 2\pi$$

$$x = \frac{5\pi}{3} + 2\pi \cdot 1 = \frac{11\pi}{3} > 2\pi$$

$$k = 2:$$

$$x = \pi \cdot 2 = 2\pi$$

Answer: The solution set is $\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi\right\}$.

$$2. \tan^2(x) - \tan(x) = 0$$

$$\tan(x)(\tan(x) - 1) = 0$$

$$\tan(x) = 0 \quad \text{or} \quad \tan(x) - 1 = 0$$

$$\tan(x) = 0 \quad \text{or} \quad \tan(x) = 1$$

$$x = \pi k, k \in Z \quad \text{or} \quad x = \frac{\pi}{4} + \pi k, k \in Z$$

Find x in the interval $0 \leq x \leq 2\pi$:

$$k = 0:$$

$$x = \pi \cdot 0 = 0$$

$$x = \frac{\pi}{4} + \pi \cdot 0 = \frac{\pi}{4}$$

$$k = 1:$$

$$x = \pi \cdot 1 = \pi$$

$$x = \frac{\pi}{4} + \pi \cdot 1 = \frac{5\pi}{4}$$

$$k = 2:$$

$$x = \pi \cdot 2 = 2\pi$$

$$x = \frac{\pi}{4} + \pi \cdot 2 = \frac{9\pi}{4} > 2\pi$$

Answer: The solution set is $\left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi\right\}$.

$$3. 2\cos^2(x) + 3\cos(x) + 1 = 0$$

$$2\cos^2(x) + 2\cos(x) + \cos(x) + 1 = 0$$

$$2\cos(x)(\cos(x) + 1) + (\cos(x) + 1) = 0$$

$$(\cos(x) + 1)(2\cos(x) + 1) = 0$$

$$\cos(x) + 1 = 0 \quad \text{or} \quad 2\cos(x) + 1 = 0$$

$$\cos(x) = -1 \quad \text{or} \quad 2\cos(x) = -1$$

$$\cos(x) = -1 \quad \text{or} \quad \cos(x) = -\frac{1}{2}$$

$$x = \pi + 2\pi k, k \in Z \quad \text{or} \quad x = \frac{2\pi}{3} + 2\pi k, k \in Z \quad \text{or} \quad x = \frac{4\pi}{3} + 2\pi k, k \in Z$$

Find x in the interval $0 \leq x \leq 2\pi$:

$$k = 0:$$

$$x = \pi + 2\pi \cdot 0 = \pi$$

$$x = \frac{2\pi}{3} + 2\pi \cdot 0 = \frac{2\pi}{3}$$

$$x = \frac{4\pi}{3} + 2\pi \cdot 0 = \frac{4\pi}{3}$$

$$k = 1:$$

$$x = \pi + 2\pi \cdot 1 = 3\pi > 2\pi$$

$$x = \frac{2\pi}{3} + 2\pi \cdot 1 = \frac{8\pi}{3} > 2\pi$$

$$x = \frac{4\pi}{3} + 2\pi \cdot 1 = \frac{10\pi}{3} > 2\pi$$

Answer: The solution set is $\left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$.