Trigonometry Proofs
(Show work)

1. sin2θcotθ = 2 – 2sin²θ

We start with the left-hand side:

$\sin(2\theta)\cot\theta = 2\sin\theta\cos\theta\dfrac{\cos\theta}{\sin\theta}$ $\quad$ $\boxed{\sin(2\theta) = 2\sin\theta\cos\theta}$, $\boxed{\cot\theta = \dfrac{\cos\theta}{\sin\theta}}$

$= 2\cos^2\theta$

$= 2(1-\sin^2\theta)$ $\quad$ $\boxed{\sin^2\theta + \cos^2\theta = 1 \quad \Rightarrow \quad \cos^2\theta = 1-\sin^2\theta}$

$= 2 - 2\sin^2\theta$

Having arrived at the right side, the identity is established.

Sketching trigonometric functions
(Show work)

1. $f(x) = 4\cos(2x + \pi)$

Comparing $f(x) = 4\cos(2x + \pi)$ to $y = A\cos(\omega t - \phi)$, we find that the amplitude is $A = 4$ $\omega = 2$ and $\phi = -\pi$. The period is

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \pi$

The graph of $f(x) = 4\cos(2x + \pi)$ will lie between –4 and 4 on the y-axis. One cycle will begin at x = 0 and end at x = π. We divide the interval [0, π] into four subintervals, each of length $\dfrac{\pi}{4}$: $\left[0,\dfrac{\pi}{4}\right]$, $\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right]$, $\left[\dfrac{\pi}{2},\dfrac{3\pi}{4}\right]$ and $\left[\dfrac{3\pi}{4},\pi\right]$.

Calculate function at the endpoints of these intervals:

$f(0) = 4\cos(2\cdot 0 + \pi) = 4\cos\pi = 4(-1) = -4$

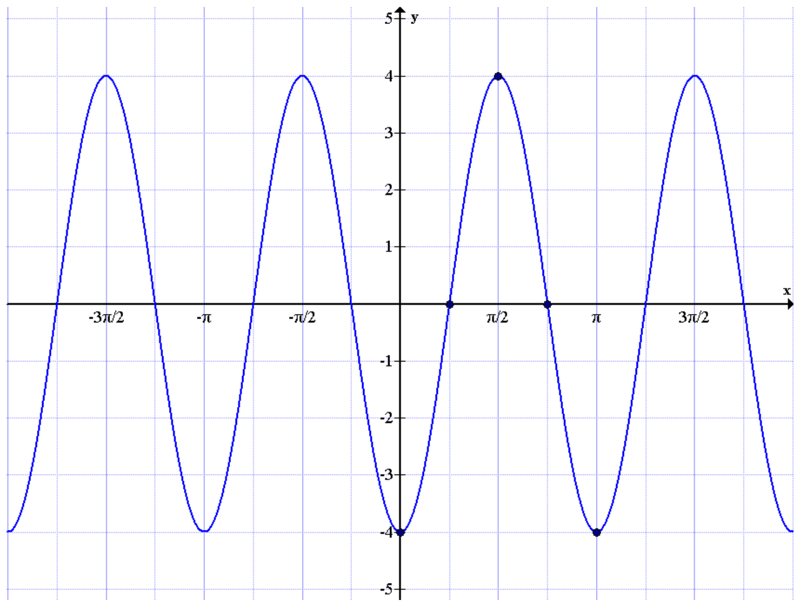
$f\left(\dfrac{\pi}{4}\right) = 4\cos\left(2\cdot\dfrac{\pi}{4} + \pi\right) = 4\cos\left(\dfrac{\pi}{2} + \pi\right) = 4\cos\left(\dfrac{3\pi}{2}\right) = 4\cdot 0 = 0$

$f\left(\dfrac{\pi}{2}\right) = 4\cos\left(2\cdot\dfrac{\pi}{2} + \pi\right) = 4\cos(\pi + \pi) = 4\cos(2\pi) = 4\cdot 1 = 4$

$f\left(\dfrac{3\pi}{4}\right) = 4\cos\left(2\cdot\dfrac{3\pi}{4} + \pi\right) = 4\cos\left(\dfrac{3\pi}{2} + \pi\right) = 4\cos\left(\dfrac{5\pi}{2}\right) = 4\cdot 0 = 0$

$f(\pi) = 4\cos(2\pi + \pi) = 4\cos(3\pi) = 4\cdot(-1) = -4$

We plot five points (0, –4), $\left(\dfrac{\pi}{4},0\right)$, $\left(\dfrac{\pi}{2},4\right)$, $\left(\dfrac{3\pi}{4},0\right)$, $(\pi,-4)$ and extend the graph in either direction.



Solving Trigonometric Equations
Find all x, such 0≤x≤2π
(Show work)

1. $\sin(2x) = \sin(x)$

$\sin(2x) - \sin(x) = 0$

$2\sin(x)\cos(x) - \sin(x) = 0$ $\qquad$ $\boxed{\sin(2x) = 2\sin(x)\cos(x)}$

$\sin(x)(2\cos(x) - 1) = 0$

$\sin(x) = 0 \quad or \quad 2\cos(x) - 1 = 0$

$\sin(x) = 0 \quad or \quad 2\cos(x) = 1$

$\sin(x) = 0 \quad or \quad \cos(x) = \frac{1}{2}$

$x = \pi k, k \in Z \quad or \quad x = \frac{\pi}{3} + 2\pi k, k \in Z \quad or \quad x = \frac{5\pi}{3} + 2\pi k, k \in Z$

Find x in the interval $0 \le x \le 2\pi$:

$k = 0$:

$x = \pi \cdot 0 = 0$

$x = \frac{\pi}{3} + 2\pi \cdot 0 = \frac{\pi}{3}$

$x = \frac{5\pi}{3} + 2\pi \cdot 0 = \frac{5\pi}{3}$

$k = 1$:

$x = \pi \cdot 1 = \pi$

$x = \frac{\pi}{3} + 2\pi \cdot 1 = \frac{7\pi}{3} > 2\pi$

$x = \frac{5\pi}{3} + 2\pi \cdot 1 = \frac{11\pi}{3} > 2\pi$

$k = 2$:
$x = \pi \cdot 2 = 2\pi$

Answer: The solution set is $\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi\right\}$.

2. tan²(x) – tan (x) = 0

$\tan(x)(\tan(x) - 1) = 0$

$\tan(x) = 0$	*or*	$\tan(x) - 1 = 0$
$\tan(x) = 0$	*or*	$\tan(x) = 1$
$x = \pi k, k \in Z$	*or*	$x = \frac{\pi}{4} + \pi k, k \in Z$

Find *x* in the interval 0≤*x*≤2π:

$k = 0$:
$x = \pi \cdot 0 = 0$
$x = \frac{\pi}{4} + \pi \cdot 0 = \frac{\pi}{4}$
$k = 1$:
$x = \pi \cdot 1 = \pi$
$x = \frac{\pi}{4} + \pi \cdot 1 = \frac{5\pi}{4}$
$k = 2$:
$x = \pi \cdot 2 = 2\pi$
$x = \frac{\pi}{4} + \pi \cdot 2 = \frac{9\pi}{4} > 2\pi$

Answer: The solution set is $\left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi\right\}$.

3. 2cos²(x) + 3cos (x) + 1 = 0

$2\cos^2(x) + 2\cos(x) + \cos(x) + 1 = 0$

$2\cos(x)(\cos(x) + 1) + (\cos(x) + 1) = 0$

$(\cos(x) + 1)(2\cos(x) + 1) = 0$

$\cos(x) + 1 = 0$	*or*	$2\cos(x) + 1 = 0$		
$\cos(x) = -1$	*or*	$2\cos(x) = -1$		
$\cos(x) = -1$	*or*	$\cos(x) = -\frac{1}{2}$		
$x = \pi + 2\pi k, k \in Z$	*or*	$x = \frac{2\pi}{3} + 2\pi k, k \in Z$	*or*	$x = \frac{4\pi}{3} + 2\pi k, k \in Z$

Find *x* in the interval 0≤*x*≤2π:

$k = 0$:

$x = \pi + 2\pi \cdot 0 = \pi$

$x = \frac{2\pi}{3} + 2\pi \cdot 0 = \frac{2\pi}{3}$

$x = \frac{4\pi}{3} + 2\pi \cdot 0 = \frac{4\pi}{3}$

$k = 1$:

$x = \pi + 2\pi \cdot 1 = 3\pi > 2\pi$

$x = \frac{2\pi}{3} + 2\pi \cdot 1 = \frac{8\pi}{3} > 2\pi$

$x = \frac{4\pi}{3} + 2\pi \cdot 1 = \frac{10\pi}{3} > 2\pi$

Answer: The solution set is $\left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$.